

Math 302, Section B1; April 2, 2003
Quiz 8

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Problem 1.

Suppose P is a point not on a line l and that \overline{PR} is a perpendicular drawn from P to l . Let d_0 be the angle of parallelism for P and l and suppose $d_0 < 90^\circ$. Let Z be such that $d_0 < m\angle RPZ < 90^\circ$.

Prove that the lines l and \overleftrightarrow{PZ} are parallel.

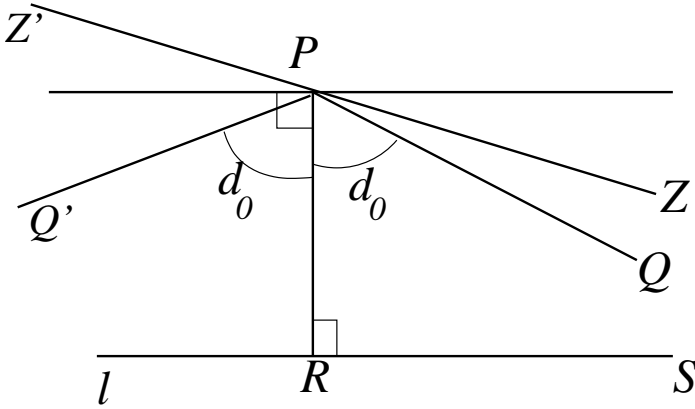


Figure 1: Here the angles $\angle RPQ$ and $\angle RPQ'$ are the angles of parallelism for P and l

Solution.

Let Z' be the point on \overleftrightarrow{PZ} such that $Z' - P - Z$. Also let Q' be the point on the same side of \overleftrightarrow{PR} as Z' such that $m\angle RPQ' = d_0 = m\angle RPQ$. By Theorem 6.2.2 $\angle RPQ'$ is the angle of parallelism for P and l .

Since $\angle RPQ$ is the angle of parallelism for P and l , by definition of the angle of parallelism we have $d_0 = m\angle RPQ \geq m\angle RPX$ for any X on the same side of \overleftrightarrow{PR} as Q such that $\overrightarrow{PX} \cap l \neq \emptyset$. Since $m\angle RPZ > d_0$, this implies that $\overrightarrow{PZ} \cap l = \emptyset$.

Now $m\angle RPZ < 90^\circ$ implies that

$$m\angle Z'PR = 180^\circ - m\angle RPZ > 90^\circ > d_0 = m\angle RPQ'.$$

Again, since $\angle RPQ'$ is the angle of parallelism for P and l and $m\angle Z'PR > m\angle RPQ'$, the definition of angle of parallelism implies that $\overrightarrow{PZ'} \cap l = \emptyset$.

Thus $\overrightarrow{PZ} \cap l = \emptyset$ and $\overrightarrow{PZ'} \cap l = \emptyset$.

Hence $\overleftrightarrow{PZ} \cap l = \emptyset$, that is l is parallel to \overleftrightarrow{PZ} .