Math 302, Section B1; April 9, 2003
Quiz 9 with Solutions

Problem 1.

(1) In hyperbolic geometry is it possible that for some Lambert quadrilateral Q the segment joining the midpoints of two opposite sides of Q is perpendicular to both of them? Explain why or why not.

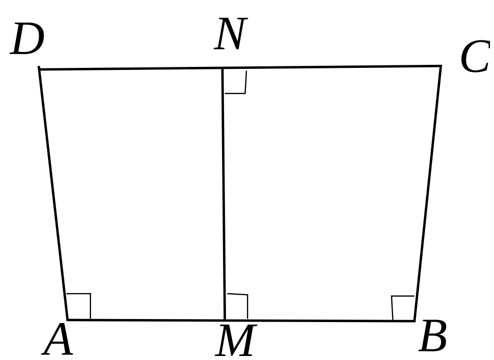
Solution.

No, it is not possible. Indeed, suppose in hyperbolic geometry there exists a Lambert quadrilateral Q such that the segment joining the midpoints of two opposite sides of Q is perpendicular to both of them. Then at least one of the two quadrilaterals, into which this segment subdivides Q, is a ractangle. This contradicts Theorem 6.3.3 which states that in hyperbolic geometry rectangles do not exist.

(2) Explain why in hyperbolic geometry there does exist a Lambert quadrilateral.

Solution.

The axioms of neutral geometry (which are included in hyperbolic geometry) guarantee that there exists a Saccheri quadrilateral. Indeed, first using SMSG Postulate 5 we can find two distinct points A and B in a plane and then using Postulate 1 draw a line $\overleftrightarrow{AB}$ through these points. Then using Postulate 12 (Angle Construction Postulate) and Postulate 9 (Plane Separation Postulate) we can construct rays with origins A and B which are perpendicular to $\overleftrightarrow{AB}$ and lie in on the same side of $\overleftrightarrow{AB}$. Then, using the Ruler Placement Postulate (SMSG Postulate 4) we can find points D and C on these rays so that $AD = AB = BC$. Finally, after connecting D and C by a segment, we obtain a Saccheri quadrilateral $ABCD$ (see Figure).



Take M to be the midpoint of $\overline{AB}$ and N to be the midpoint of $\overline{CD}$. By Theorem 3.6.4 the segment $\overline{MN}$ is perpendicular to both $\overline{AB}$ and $\overline{CD}$. Therefore $MNCB$ is a Lambert quadrilateral since the angles at M, N and B are right.