

Math 317 Section B1 Quiz 7 (with solutions)

March 15, 2002

**Problem 1.**

Let  $G = S_n$  where  $n \geq 3$ . Consider the function  $f : G \rightarrow G$  defined as  $f(\sigma) = (123)\sigma(123)$  for every  $\sigma \in G$ .

Is  $f$  a homomorphism? Give a detailed justification of your answer.

**Solution.**

No,  $f$  is not a homomorphism, since  $f$  does not respect multiplication. For example,  $1 \cdot 1 = 1$  and  $f(1) = (123)(123) = (321)$  but

$$f(1) \cdot f(1) = (321)(321) = (123) \neq (321) = f(1).$$

Moreover  $f(1) = (321) \neq 1$ , which provides another reason why  $f$  is not a homomorphism.