Math 415 Challenge of the week

September 21, 2001

Solutions of these proiblems will be discussed on Friday, September 28.

Problem 1.

We say that a geodesic n-gon in a metric space X is K-*slim* if each side of this n-gon is contained in the K-neighborhood of the union of the other $(n-1)$ sides. Thus in a δ-hyperbolic space all geodesic 3-gons (triangles) are δ-thin.

Find a function $f(n)$ (as small as possible), where $n \geq 3$, such that in any δ-hyperbolic geodesic metric space (X, d) every geodesic n-gon is $\delta f(n)$-slim.

[**Hint:** Think about the subdivision trick in the proof that geodesics in a hyperbolic metric space diverge exponentially]

Problem 2.

Show that hyperbolicity of the Gromov product is NOT a quasi-isometry invariant. That is, find quasi-isometric metric spaces X and Y such that for some $x \in X$ and $\delta \geq 0$ the Gromov product $(-,-)_x$ in X is δ-hyperbolic and such that for every $y \in Y$ and every $\delta' \geq 0$ the Gromov product $(-,-)_y$ in Y is not δ'-hyperbolic.