

Quiz 2; Friday, February 4 (SOLUTIONS)

(1) Prove that $17|(2 + 100^{17})$.

Solution. Since 17 is a prime, Fermat's Little Theorem implies that

$$100^{17} \equiv 100 \pmod{17}.$$

We have $100 = -2 + 102 = -2 + 6 \cdot 17$. Hence

$$100^{17} \equiv 100 \pmod{17} \equiv -2 \pmod{17}$$

and therefore $17|(2 + 100^{17})$.

(2) Let $a, b \in \mathbb{Z}$ be such that $3a + 5b = 1$ and such that a is odd. Find $\gcd(a, 20)$.

Solution.

Since $3a + 5b = 1$, and $5 \nmid 1$, it follows that $5 \nmid a$. Indeed, if $5|a$ then $5|(3a + 5b) = 1$, yielding a contradiction.

By assumption a is odd, so $2 \nmid a$. We have $20 = 2^2 \cdot 5$ and none of the primes occurring in the prime factorization of 20 divide a . Therefore $\gcd(a, 20) = 1$.