

### Quiz 3 (Solutions); Friday, February 18

Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 5 & 9 & 4 & 7 & 8 & 2 & 3 & 6 \end{pmatrix} \in S_9.$$

Let  $\alpha = (39)\sigma(12)\sigma^{-1}(29)(45)(14)\sigma \in S_9$ .

(a) Find a complete factorization of  $\sigma$ .

(b) Find  $\text{sgn}(\alpha)$ .

#### Solution

(a) The complete factorization of  $\sigma$  is

$$\sigma = (257)(3968)(1)(4).$$

(b) The complete factorization of  $\sigma$  involves  $t = 4$  cycles. Therefore by definition  $\text{sgn}(\sigma) = (-1)^{9-4} = (-1)^5 = -1$ . Since  $\text{sgn}(\sigma^{-1}) = \text{sgn}(\sigma) = -1$  and transpositions have signum  $-1$ , by the multiplicativity of signum we get

$$\begin{aligned} \text{sgn}(\alpha) &= \\ \text{sgn}((39))\text{sgn}(\sigma)\text{sgn}((12))\text{sgn}(\sigma^{-1})\text{sgn}((29))\text{sgn}((45))\text{sgn}((14))\text{sgn}(\sigma) &= \\ (-1)^8 &= 1. \end{aligned}$$