# Quiz 6 (Solution); Friday, March 7, 2008

**Problem 1.**
Prove that the groups $\mathbb{Z}_4 \times \mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_8$ are not isomorphic.

**Solution.**
For every element $a \in \mathbb{Z}_4$ we have $|a| \in \{1, 2, 4\}$, so that $|a| \big| 4$. For an arbitrary $(a, b) \in \mathbb{Z}_4 \times \mathbb{Z}_4$ we have $|(a, b)| = lcm(|a|, |b|)$. Since both $|a|$ and $|b|$ divide 4, it follows that $|(a, b)| = lcm(|a|, |b|)$ also divides 4 and this is $\leq 4$. In particular, the group $\mathbb{Z}_4 \times \mathbb{Z}_4$ has no elements of order 8.

On the other hand, for $g = ([0]_2, [1]_8) \in \mathbb{Z}_2 \times \mathbb{Z}_8$ we have $|g| = lcm(1, 8) = 8$. Thus $\mathbb{Z}_2 \times \mathbb{Z}_8$ has an element of order 8 while $\mathbb{Z}_4 \times \mathbb{Z}_4$ has no elements of order 8. Therefore the groups $\mathbb{Z}_4 \times \mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_8$ are not isomorphic.