

Extra Credit Problem Set 3; Due Wednesday, May 6, 2009

Print your name:

1. Let G be a group and R be a ring. Consider the set RG as

$$RG = \left\{ \sum_{g \in G} a_g g \mid a_g \in R, \text{ and } a_g = 0 \text{ for all but finitely many } g \in G \right\}$$

of formal finite R -linear combinations of elements of G .

For $f_1 = \sum_{g \in G} a_g g$ and $f_2 = \sum_{g \in G} b_g g$ define

$$f_1 + f_2 := \sum_{g \in G} (a_g + b_g)g,$$

$$f_1 \cdot f_2 := \sum_{g \in G} c_g g,$$

where for every $g \in G$ we have $c_g = \sum_{h \in G} a_h b_{h^{-1}g}$.

Prove that $(RG, +, \cdot)$ is a ring.

[This ring is called the *group ring of G over R* .]

2. Let G be a finite group and let $p \geq 3$ be a prime such that $p \nmid |G|$.

Prove that the group ring $\mathbb{Z}_p G$ is not a domain.

Hint: Think about the value of $(g - 1)^p$ in $\mathbb{Z}_p G$ where $g \in G$ and where $1 = e \in G$ is the identity element of G .