

### Quiz 3 (Solutions); Friday, February 13

1. Let  $\sigma = (1\ 2\ 5\ 6)(2\ 3\ 4)(1\ 6)(2\ 4) \in S_6$ .

(a) Compute  $\sigma$  in a two-row notation and find a decomposition of  $\sigma$  as a product of disjoint cycles.

(b) Find the parity of  $\sigma$  and the parity of  $\sigma^{75}$ . Justify your answers.

**Solution.**

(a) We have

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 5 & 4 & 3 & 6 & 2 \end{pmatrix} = (2\ 5\ 6)(3\ 4).$$

(b) A 2-cycle  $(3\ 4)$  is odd and a 3-cycle  $(2\ 5\ 6)$  is even hence  $\sigma = (2\ 5\ 6)(3\ 4)$  is odd. Since  $\sigma$  is odd, 75 is an odd integer and the product of an odd number of odd permutations is odd, it follows that  $\sigma^{75}$  is odd.