

Quiz 5 (Solutions); Friday, March 6, 2009

1. For each of the following statements indicate if it is true or false. You do not need to explain your answers here.

- (1) The element $\overline{18} \in \mathbb{Z}_{20}$ is a generator of \mathbb{Z}_{20} .
- (2) If $g \in G$ and $|g| = 36$ then $|g^4| = 4$.
- (3) If G is a cyclic group then every subgroup of G is abelian.
- (4) If $a, b \in G$ are such that $|\langle a \rangle| = |\langle b \rangle|$ then $a^{|b|} = 1_G$.
- (5) If $G = \langle a \rangle$ is a cyclic group generated by a then $\langle a^5, a^4 \rangle = G$.

Answers:

- (1) False. Since $\gcd(18, 20) = 2 \neq 1$, it follows that $\overline{18}$ is not a generator of \mathbb{Z}_{20} .
- (2) False. We have $|g^4| = \frac{36}{4} = 9$.
- (3) True. A cyclic group is abelian and every subgroup of an abelian group is abelian.
- (4) True. We know that the order of a cyclic group generated by an element is equal to the order of that element. Therefore a and b have equal orders and hence $a^{|b|} = 1_G$.
- (5) True. We have $a^5 \cdot a^{-4} = a \in \langle a^5, a^4 \rangle$ and so $G = \langle a \rangle \subseteq \langle a^5, a^4 \rangle \subseteq G$. Therefore $G = \langle a^5, a^4 \rangle$.