

## Quiz 8 (Solutions); Friday, April 3, 2009

1. For each of the following statements indicate if it is true or false. You do not need to explain your answers here.

- (1) If  $G$  is an abelian group and  $H \leq G$  is a subgroup then the factor group  $G/H$  is abelian.
- (2) If  $G$  is a non-abelian group and  $H \triangleleft G$  is a normal subgroup then the factor group  $G/H$  is non-abelian.
- (3) If  $H \triangleleft G$  is a normal subgroup of index  $p \geq 2$  where  $p$  is a prime, then the quotient group  $G/H$  is cyclic.
- (4) If  $G$  and  $H$  are groups and  $G_1 := \{(g, 1_H) : g \in G\}$  then  $G_1$  is a normal subgroup of  $G \times H$ .
- (5) The subgroup  $H = \langle(1\ 2\ 3)\rangle \leq S_4$  is normal in  $S_4$ .

### Answers:

(1) True.

(2) False. Indeed, for any group  $G$  we have  $G \triangleleft G$ . The factor group  $G/G = \{1G\}$  is the trivial group in this case and thus abelian. However,  $G$  can be chosen to be nonabelian, e.g.  $G = S_3$ .

(3) True. The factor group  $G/H$  is of order  $p$  and since  $p$  is a prime, the group  $G/H$  is cyclic by Corollary 3 in Ch. 2.6.

(4) True.

(5) False. E.g.

$$(1\ 4)(1\ 2\ 3)(1\ 4)^{-1} = (1\ 4)(1\ 2\ 3)(1\ 4) = (4\ 2\ 3) \notin \langle(1\ 2\ 3)\rangle.$$

Hence  $\langle(1\ 2\ 3)\rangle$  is not normal in  $S_4$ .