

Math 481
1. Examples of Manifolds

1. Examples of manifolds:

- a) S^1 or S^2 = configuration space of a pendulum in the plane or 3-space.
- b) $T^2 = S^1 \times S^1$ = configuration space of a double pendulum in the plane (superposition allowed).
- c) P^1 or P^2 = configuration space of a rod with fixed centerpoint (ends not distinguished) in the plane or 3-space.
- d) Note: Configuration space of a rod with left endpoint fixed OR right endpoint fixed is NOT a manifold:

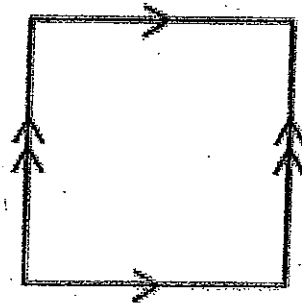
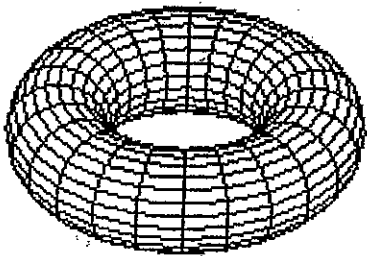


- e) $S^n = \{\vec{x} \in E^{n+1} : |\vec{x}| = 1\}$
- f) $T^n = S^1 \times \dots \times S^1$ (n times).
- g) $P^n = S^n / \{\vec{x} = -\vec{x}\}$.

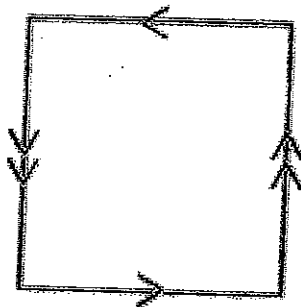
Question: Can one distinguish between P^1 and S^1 ? Between P^2 and S^2 ?

2. Models of manifolds:

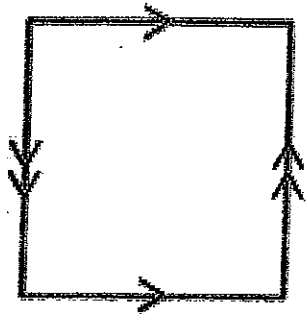
- a) T^2 :



- b) P^2 :

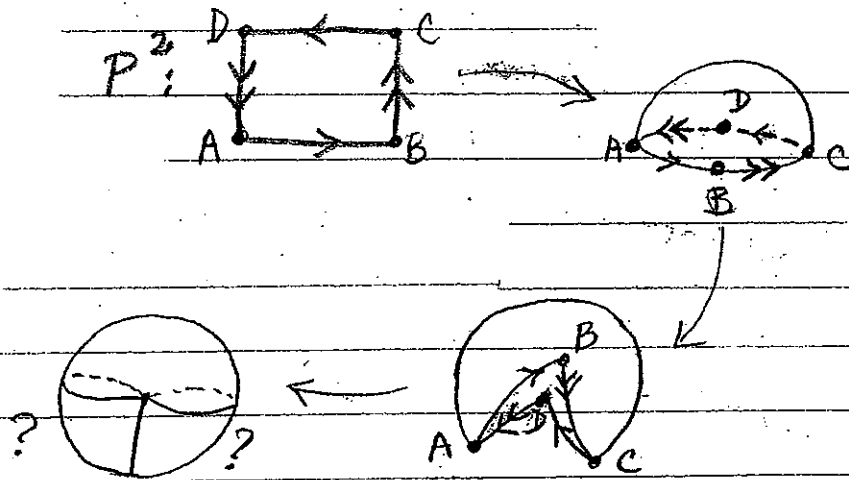


c) Klein Bottle:

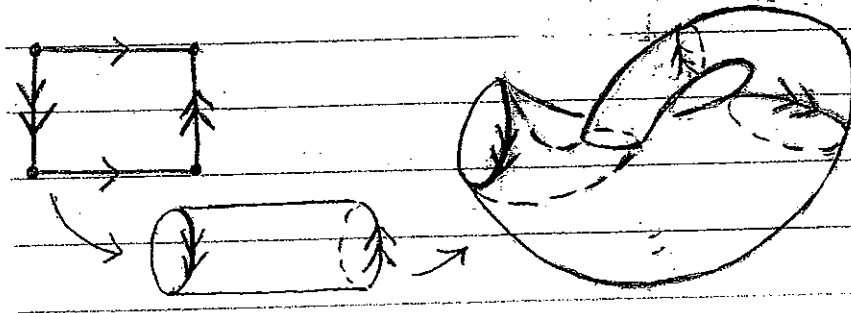


3.

a) Can P^2 be realized as a surface in E^3 ?



b) Can the Klein bottle be realized as a surface in E^3 ?

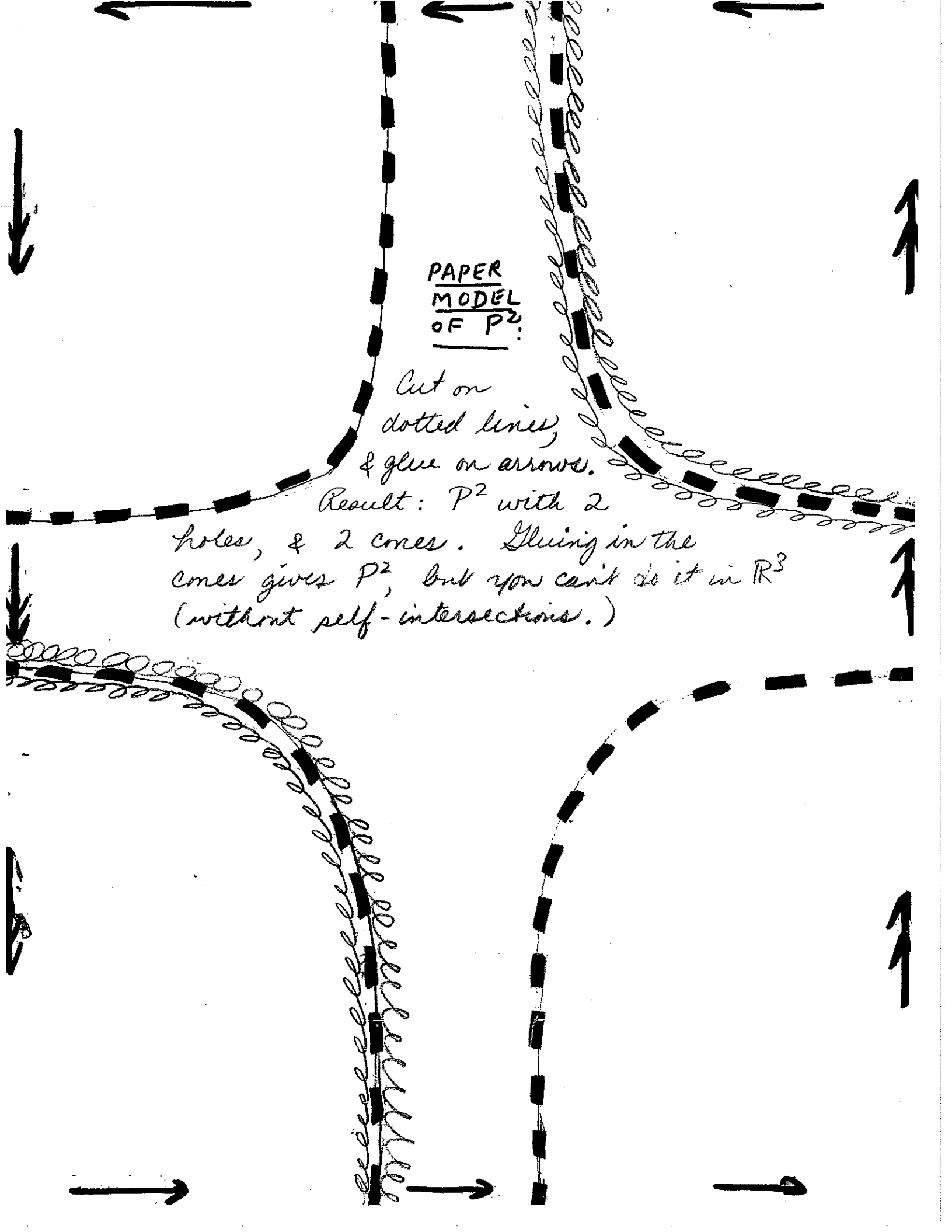


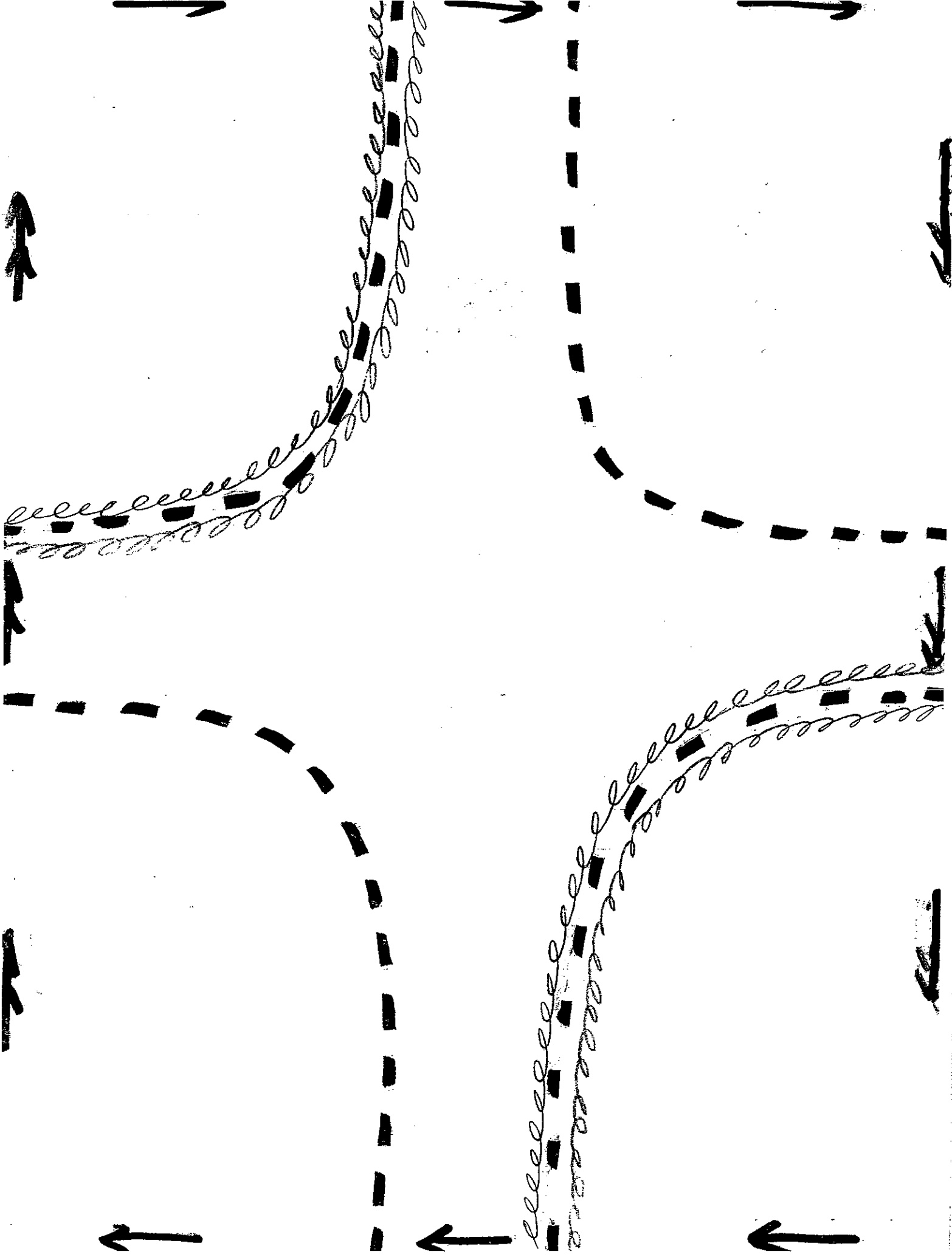
Read: Frankel, pp. 1, 11-16.

PAPER
MODEL
OF P^2 .

Cut on
dotted lines,
& glue on arrows.

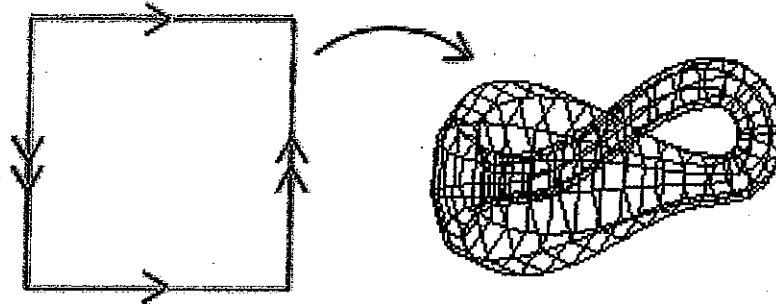
Result: P^2 with 2
holes, & 2 cones. Gluing in the
cones gives P^2 , but you can't do it in \mathbb{R}^3
(without self-intersections.)





2. Definition of Differentiable Manifold

1. a) The Klein Bottle, K^2 .



- b) K^2 is *not* homeomorphic to the space on the right since the self-intersection should not be there. In fact, K^2 can *not* be embedded in \mathbb{R}^3 . It can in \mathbb{R}^4 : how?
2. a) In the text, §1.2a, on Topology: Spaces are *homeomorphic* (= *topologically equivalent*) if there is a one-to-one map f of M onto N such that f and f^{-1} are continuous.
- b) In this course, our spaces are *manifolds*. These are locally homeomorphic to an open set in \mathbb{R}^n (see below).
- c) Examples of glued spaces that are/are not manifolds?
3. a) In the text, §1.2 b & c, on Definition of an n -dimensional manifold M : A *coordinate patch* is a map $\phi_U : U \rightarrow \mathbb{R}^n$ (where $U \subset M$) which is a one-to-one map onto an open set in \mathbb{R}^n .
- b) An *atlas* is a collection ϕ_{U_i} , where $M = \cup_i U_i$, and any two patches are *compatible* (see 4. c) below) when they overlap.
4. a) Example: An atlas on S^1 . Take

$$(U = S^1 - (0, 1), \phi_U), (V = S^1 - (0, -1), \phi_V)$$

where ϕ_U, ϕ_V satisfy

$$\begin{aligned} \frac{\phi_U(p)}{2} &= \frac{x}{1-y} \\ \frac{\phi_V(p)}{2} &= \frac{x}{1+y} \\ \phi_U(p)\phi_V(p) &= \frac{4x^2}{1-y^2} = 4 \end{aligned}$$

(see figure, over).

