

Exam 1 Information and Review - Math 221, Sections DL1, Fall 2008

- This exam will cover sections 1.2-1.5, 2.1-2.7.
- Wednesday, September 24, 314 Altgeld Hall, 1:00-1:50
- **Please bring your I-card to the exam.**
- No calculators, books, or notes allowed. You can not use any headphones during the exam.
- Seating will be assigned when you enter the room.
- Anyone who has exam accommodations through DRES, you must contact DRES by Sept. 17.

The review problems below represent kinds of problems that may appear on the exam. They do *not* include every possible topic that may be covered on the exam. Also, the exam will be quite a bit shorter than this review page! Re-doing homework problems without looking at your book or notes is also good practice.

1. For each part, answer true (T) or false (F). No explanation is needed and there is no partial credit.
  - (a) Every function has a derivative.
  - (b) Every continuous function has a derivative.
  - (c)  $\lim_{x \rightarrow \infty} \sin x$  does not exist.
  - (d) Instantaneous velocity is always positive.
  - (e) Instantaneous velocity is the derivative of the position function.
  - (f)  $f(x) = x^{1/3}$  is differentiable for all  $x$ .
  - (g) If  $f(a)$  is undefined, then  $f$  is not differentiable at  $x = a$ .
  - (h) If  $f$  is differentiable at  $x = a$ , then  $f$  is continuous at  $x = a$  (see page 163).
  - (i) If  $\lim_{x \rightarrow a} f(x) = \infty$  and  $\lim_{x \rightarrow a} g(x) = -\infty$ , then  $\lim_{x \rightarrow a} [f(x) + g(x)] = 0$ .
  - (j) If  $\lim_{x \rightarrow 1} f(x) = 4$ , then  $f(1) = 4$ .
  - (k) If  $f(1) = 4$ , then  $\lim_{x \rightarrow 1} f(x) = 4$ .
2. Give an example of each of the following. The example can be given as a formula or as a graph as long as the relevant features of the graph are very clearly drawn so there is no ambiguity about what is meant:
  - (a) A function with a removable discontinuity at  $x = 0$ .
  - (b) A function with a non-removable discontinuity at  $x = 0$ .
  - (c) A function  $f(x)$  for which  $\lim_{x \rightarrow 3^+} f(x) \neq \lim_{x \rightarrow 3^-} f(x)$ .
  - (d) A function which is continuous for all  $x$ .
  - (e) A function for which  $\lim_{x \rightarrow 0} f(x) = -\infty$ .
  - (f) A function which is not differentiable at  $x = 4$ .
  - (g) A function for which  $f''(x) = 0$  for all  $x$ .
  - (h) A function for which  $f'(-1) = 0$  and  $f(1) = 1$ .

- (i) A function  $f(x)$  with the property  $f'(x) < 0$  for all  $x$ .
- (j) A function  $f(x)$  with the property  $f'(x) = f(x)$  for all  $x$ .
3. Find the derivative of each of the following (you can use derivative rules - you do not have to use the definition of derivative for these).
- (a)  $f(x) = 3\sqrt{x}$
- (b)  $f(x) = x^2 \sin x$
- (c)  $f(x) = \frac{\ln x}{\cos x^2}$
- (d)  $f(x) = \sec(e^{5x})$
- (e) For many more practice problems on derivatives and higher order derivatives, see Review Exercises #23-46, 53-60 on pages 236-7. Answers to the odd-numbered problems are in the back of the book. You can also use the “Practice Exercises” on MathZone for each section. Go to the “Self Study” tab to find these.
4. Evaluate each limit. Answer with a number,  $\infty$ ,  $-\infty$ , or “does not exist.”

(a)

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + x - 2}$$

(b)

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$$

(c)

$$\lim_{x \rightarrow 1} e^{x^2 + x - 2}$$

(d)

$$\lim_{x \rightarrow \pi^-} \csc x$$

(e)

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x}$$

(f)

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 4x + 8}{3x^3}$$

(g)

$$\lim_{x \rightarrow 0^+} \frac{x^2 - 4x + 8}{3x^3}$$

(h)

$$\lim_{x \rightarrow 0} \frac{x}{|x|}$$

- (i) For many more practice problems, see Review Exercises #15-35 on page 142, also MathZone “Practice Exercises”.

5. State the Squeeze Theorem.

6. Suppose we have a function  $g(x)$  and we know that  $5 - 2x^2 \leq g(x) \leq 5 - x^2$  for all  $x$  between  $-1$  and  $1$ . Explain how the Squeeze Theorem can be used to find  $\lim_{x \rightarrow 0} g(x)$ .
7. Find the slope of the secant line to  $f(x) = x^2 - x$  from  $x = 1$  to  $x = 1 + h$ . Then take the limit as  $h \rightarrow 0$  to find the slope of the tangent line to  $f(x) = x^2 - x$  at  $x = 1$ .
8. State the definition of the derivative of a function  $f(x)$  at a point  $x = a$ .
9. Use the definition of the derivative to show that  $f'(1) = 3$  for  $f(x) = x^3$ .
10. Given the graph of  $f$ , sketch the graph of  $f'$ , and given the graph of  $f'$ , sketch the graph of  $f$ . See exercises 13-24 from Section 2.2, also #41 from Section 2.3.
11. Suppose you know that  $f(1) = 2$ ,  $f'(1) = 4$ ,  $f'(2) = 3$ , and  $f$  has an inverse  $f^{-1}$ . Find  $\frac{d}{dx} f^{-1}(2)$ .
12. Find the equation of the tangent line to  $y = \cos x$  at  $x = \pi/4$ .
13. Find  $\frac{d}{dx} x^x$ . Hint: logarithmic differentiation.