Section 2.5 5, 7, 9, 15, 19, 21, 23, 25, 27, 33, 35, 39, 51

5. $f(x) = \sqrt{x^2+4}$

$$f'(x) = \frac{1}{2}(x^2+4)^{-1/2} \cdot 2x = \boxed{x(x^2+4)^{-1/2}}$$

7. $f(x) = x^5\sqrt{x^3+2}$

$$f'(x) = 5x^4\sqrt{x^3+2} + \frac{1}{2} \cdot x^5 (x^3+2)^{-1/2} \cdot 3x^2 = \boxed{5x^4\sqrt{x^3+2} + \frac{3}{2}x^7(x^3+2)^{-1/2}}$$

9. $f(x) = \dfrac{x^3}{(x^2+4)^2}$

$$f'(x) = \frac{(x^2+4)^2 \cdot 3x^2 - 2x^3(x^2+4) \cdot 2x}{(x^2+4)^4} = \frac{3x^2(x^2+4)^2 - 4x^3(x^2+4)}{(x^2+4)^4}$$

$$= \boxed{\frac{3x^2(x^2+4) - 4x^3}{(x^2+4)^3}}$$

15. $f(x) = \left(\sqrt{x^3+2} + 2x\right)^{-2}$

$$f'(x) = \boxed{-2\left(\sqrt{x^3+2}+2x\right)^{-3} \cdot \left(\frac{1}{2}(x^3+2)^{-1/2} \cdot 3x^2 + 2\right)}$$

19. $f(x) = \sqrt{\dfrac{x}{x^2+1}}$

$$f'(x) = \frac{1}{2}\left(\frac{x}{x^2+1}\right)^{-1/2} \cdot \left(\frac{x^2+1 - x(2x)}{(x^2+1)^2}\right) = \frac{1}{2}\left(\frac{x^2+1}{x}\right)^{1/2} \frac{1-x^2}{(x^2+1)^2} = \boxed{\frac{1-x^2}{2x^{1/2}(x^2+1)^{3/2}}}$$

21. $f(x) = \sqrt[3]{x\sqrt{x^4+2x\sqrt[4]{\frac{8}{x+2}}}}$

$$f'(x) = \frac{1}{3}\Big(\underbrace{x\sqrt{x^4+2x\sqrt[4]{\tfrac{8}{x+2}}}}_{g(x)}\Big)^{-2/3} \cdot \left(g'(x)\right)$$

$$g'(x) = \sqrt{x^4+2x\sqrt[4]{\frac{8}{x+2}}} + x\left(\frac{1}{2}\Big(\underbrace{x^4+2x\sqrt[4]{\tfrac{8}{x+2}}}_{h(x)}\Big)^{-1/2} \cdot \left(h'(x)\right)\right)$$

$$h'(x) = 4x^3 + 2\sqrt[4]{\frac{8}{x+2}} + 2x \cdot \cancel{\tfrac{1}{4}}\left(\frac{8}{x+2}\right)^{-3/4} \cdot \left(\frac{-\cancel{8}^{\,2}}{(x+2)^2}\right)$$, so

$$f'(x) = \boxed{\frac{1}{3}\left(x\sqrt{x^4+2x\sqrt[4]{\frac{8}{x+2}}}\right)^{-2/3}\left(\sqrt{x^4+2x\sqrt[4]{\frac{8}{x+2}}} + \frac{x}{2}\left(x^4+2x\sqrt[4]{\frac{8}{x+2}}\right)^{-1/2} \cdot \left(4x^3+2\sqrt[4]{\frac{8}{x+2}} - 4x\left(\frac{8}{x+2}\right)^{-3/4}\frac{1}{(x+2)^2}\right)\right)}$$

23 $f(x) = \sqrt{x^2+16}$, $a=3$

$$f'(x) = \frac{x}{\sqrt{x^2+16}} \qquad f(3) = \sqrt{25} = 5$$

$$f'(3) = \frac{3}{5}$$

$$\boxed{y-5 = \frac{3}{5}(x-3)}$$

25 $s(t) = \sqrt{t^2+8}$

$$v(t) = s'(t) = \frac{t}{\sqrt{t^2+8}} \qquad v(2) = \frac{2}{\sqrt{12}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

27 $f(x) = \sqrt{2x+1}$

$$f'(x) = \frac{1}{\sqrt{2x+1}} = (2x+1)^{-1/2}$$

$$f''(x) = -\frac{1}{2}\cdot 2(2x+1)^{-3/2} = -(2x+1)^{-3/2}$$

$$f'''(x) = \frac{+3}{2}\cdot 2(2x+1)^{-5/2} = 3(2x+1)^{-5/2}$$

$$f^{(4)}(x) = \frac{-3\cdot 5}{2}\cdot 2(2x+1)^{-7/2} = 3\cdot 5(2x+1)^{-7/2}$$

So

$$\boxed{f^{(n)}(x) = (-1)^{n+1}\, 1\cdot 1\cdot 3\cdot 5\cdots(2n-3)(2x+1)^{-\left(\frac{2n-1}{2}\right)}}$$

33 $h(x) = f(g(x))$

$$h'(x) = f'(g(x))\cdot g'(x) \text{ , so } h'(1) = f'(g(1))\cdot g'(1)$$

$$= f'(2)\cdot -2 = 3\cdot -2 = \boxed{-6}$$

35 $f(x) = x^3+4x-1$, $a=-1$ $\Longrightarrow$ $f'(x) = 3x^2+4$, $f(0) = -1$

Let $g(x)$ be the inverse of $f(x)$, so

$$g'(x) = \frac{1}{f'(g(x))} \text{ , } g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(0)} = \boxed{\frac{1}{4}}$$

(39) $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}$, $a = 3$ $\Rightarrow f'(x) = \frac{1}{2}(x^5 + 4x^3 + 3x + 1)^{-1/2} \cdot (5x^4 + 12x^2 + 3)$

Let $g(x)$ be the inverse of $f(x)$, so $f(1) = 3$

$$g'(x) = \frac{1}{f'(g(x))}, \text{ so } g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \frac{20}{6} = \boxed{\frac{10}{3}}$$

(51) $f(x) = b(a(x))$, so $f'(x) = b'(a(x)) \cdot a'(x)$

$$\boxed{f'(2) = b'(a(2)) \cdot a'(2) \approx -1 \cdot 4 = -4}$$

$a(2) = 0$

$a'(2) \approx \frac{4-0}{3-2} = 4$

$b'(0) \approx -1$

Section 2.6 3, 5, 7, 9, 11, 17, 19, 25, 27, 37, 43

(3) $f(x) = 4\sin x - x$

$f'(x) = \boxed{4\cos x - 1}$

(5) $f(x) = \tan^3 x - \csc^4 x$

$f'(x) = 3\tan^2 x \sec^2 x + 4\csc^3 x \csc x \cot x = \boxed{3\tan^2 x \sec^2 x + 4\csc^4 x \cot x}$

(7) $f(x) = x\cos(5x^2)$

$f'(x) = \cos(5x^2) + x \cdot -\sin(5x^2) \cdot 10x = \boxed{\cos(5x^2) - 10x^2\sin(5x^2)}$

(9) $f(x) = \sin(\tan(x^2))$

$f'(x) = \boxed{\cos(\tan(x^2)) \cdot \sec^2(x^2) \cdot 2x}$

(11) $f(x) = \dfrac{\sin(x^2)}{x^2}$

$f'(x) = \dfrac{x^2 \cdot \cos(x^2) \cdot 2x - 2x\sin(x^2)}{x^4} = \dfrac{2x^3\cos(x^2) - 2x\sin(x^2)}{x^4} = \boxed{\dfrac{2x^2\cos(x^2) - 2\sin(x^2)}{x^3}}$

(17) $f(x) = 2\sin x\cos x$

$f'(x) = \boxed{2(\cos^2 x - \sin^2 x)}$

(19) $f(x) = \tan\sqrt{x^2+1}$

$f'(x) = \boxed{\sec^2(\sqrt{x^2+1}) \cdot \dfrac{x}{\sqrt{x^2+1}}}$

(25) $f(x) = \sin 4x, \ a = \frac{\pi}{8}$ $f(\frac{\pi}{8}) = \sin\frac{\pi}{2} = 1$

$f'(x) = 4\cos 4x$ $f'(\frac{\pi}{8}) = 4\cos\frac{\pi}{2} = 0$

$\boxed{y = 1}$

(27) $f(x) = \cos x$, $a = \frac{\pi}{2}$ $\quad f(\frac{\pi}{2}) = 0$

$f'(x) = -\sin x$ $\quad f'(\frac{\pi}{2}) = -1$

$$\boxed{y = -(x - \tfrac{\pi}{2})}$$

(37) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$ $\leftarrow$ repeats every 4

$75 = 18 \cdot 4 + 3$, so

$$\boxed{f^{(75)}(x) = -\cos x}$$

$150 = 37 \cdot 4 + 2$, so

$$\boxed{f^{(150)}(x) = -\sin x}$$

(43) a) $\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} 3 \cdot \frac{\sin 3x}{3x} = 3$

b) $\lim_{x \to 0} \frac{\sin t}{4t} = \lim_{x \to 0} \frac{1}{4} \cdot \frac{\sin t}{t} = \frac{1}{4}$

c) $\lim_{x \to 0} \frac{\cos x - 1}{5x} = \lim_{x \to 0} \frac{1}{5} \cdot \frac{\cos x - 1}{x} = 0$

d) $\lim_{x \to 0} \frac{\sin x^2}{x^2} = 1$

Section 2.7 1, 3, 5, 11, 15, 17, 19, 21, 23, 25, 29, 39, 41, 43

(1) $f(x) = x^3e^x$, $f'(x) = \boxed{3x^2e^x + x^3e^x}$

(3) $f(x) = x + 2^x$

$f'(x) = \boxed{1 + 2^x \ln 2}$

(5) $f(x) = 2e^{4x+1}$

$f'(x) = 2 \cdot 4e^{4x+1} = \boxed{8e^{4x+1}}$

(11) $f(x) = \dfrac{e^{4x}}{x}$

$f'(x) = \boxed{\dfrac{4xe^{4x} - e^{4x}}{x^2}}$

(15) $f(x) = \ln(x^3 + 3x)$

$f'(x) = \boxed{\dfrac{3x^2 + 3}{x^3 + 3x}}$

(17) $f(x) = \ln(\cos x)$

$f'(x) = \dfrac{-\sin x}{\cos x} = \boxed{-\tan x}$

(19) $f(x) = \sin[\ln(\cos(x^3))]$

$f'(x) = \boxed{\cos[\ln(\cos(x^3))] \cdot \dfrac{-3x^2 \sin(x^3)}{\cos(x^3)}}$

21) $f(x) = \frac{\sqrt{\ln x^2}}{x} = \frac{\sqrt{2}\sqrt{\ln x}}{x}$

$f'(x) = \sqrt{2}\left(\frac{\cancel{x} \cdot \frac{1}{2}(\ln x)^{-1/2} \cdot \frac{1}{\cancel{x}} - \sqrt{\ln x}}{x^2}\right) = \boxed{\sqrt{2}\left(\frac{1}{2x^2\sqrt{\ln x}} - \frac{\sqrt{\ln x}}{x^2}\right)}$

23) $f(x) = \ln(\sec x + \tan x)$

$f'(x) = \boxed{\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}}$

25) $f(x) = 3e^x$, $x=1$ $\quad f(1) = 3e$

$f'(x) = 3e^x$ $\quad f'(1) = 3e$

$\boxed{y - 3e = 3e(x-1)}$

29) $f(x) = x^2 \ln x$, $x=1$ $\quad f(1) = 0$

$f'(x) = 2x\ln x + x$ $\quad f'(1) = 1$

$\boxed{y = x - 1}$

39) $f(x) = x^{\sin x}$

$\ln f(x) = \sin x \ln x$

$\frac{f'(x)}{f(x)} = \cos x \ln x + \frac{\sin x}{x}$

$f'(x) = f(x)\left(\cos x \ln x + \frac{\sin x}{x}\right) = \boxed{x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)}$

41. $f(x) = (\sin x)^x$

$$\ln f(x) = x \ln(\sin x)$$

$$\frac{f'(x)}{f(x)} = \ln(\sin x) + \frac{x \cos x}{\sin x}$$

$$f'(x) = f(x)\left(\ln(\sin x) + x \cot x\right) = \boxed{(\sin x)^x \left(\ln(\sin x) + x \cot x\right)}$$

43. $f(x) = x^{\ln x}$

$$\ln f(x) = \ln x \cdot \ln x = (\ln x)^2$$

$$\frac{f'(x)}{f(x)} = \frac{2 \ln x}{x}$$

$$f'(x) = f(x)\left(\frac{2 \ln x}{x}\right) = \boxed{2x^{\ln x - 1} \cdot \ln x}$$