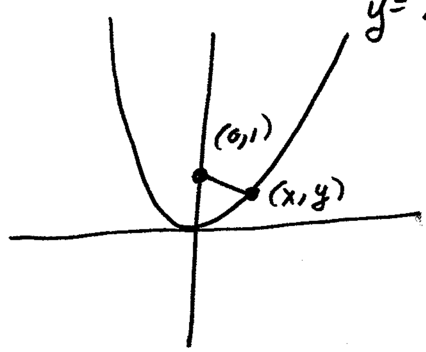
3.7 #9



Minimize distance from (x,y) to $(0,1)$

$$D = \sqrt{(y-1)^2 + (x-0)^2}$$

The same point will minimize

$D^2 = (y-1)^2 + x^2$ and is easier to work with.

$y = x^2$, so $D^2 = (y-1)^2 + y$ $\qquad 0 \le y < \infty$

$$\frac{d}{dy} D^2 = 2(y-1) + 1 = 2y - 1$$

Critical point $y = \frac{1}{2}$.

as $y \to \infty$, $D^2 \to \infty$ and when $y = 0$, $D^2 = 1$.

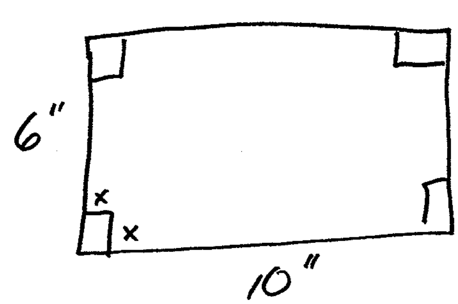
when $y = \frac{1}{2}$, $D^2 = \frac{3}{4}$. Therefore the abs. minimum occurs when $y = \frac{1}{2}$

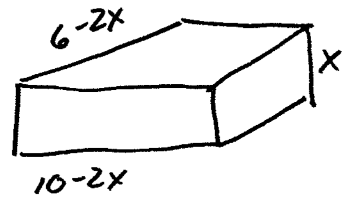
To find the x-coordinate

$$y = x^2$$
$$\frac{1}{2} = x^2$$
$$\frac{\pm 1}{\sqrt{2}} = x$$

Answer: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

3.7 #15





Volume $= x(6-2x)(10-2x)$ ~~Minimi~~ Maximize.

$0 \le x \le 3$

$V = 4x(3-x)(5-x)$

$V' = 4(3-x)(5-x) + 4x(-1)(5-x) + 4x(3-x)(-1)$

$= 4(15-8x+x^2) - (20x-4x^2) \cancel{+} (12x-4x^2)$

$= 60 - 32x + 4x^2 - 20x + 4x^2 - 12x + 4x^2$

$= 12x^2 - 64x + 60$

$= 4(3x^2 - 16x + 15)$

To find critical points, use quadratic formula

$$x = \frac{16 \pm \sqrt{256-180}}{6} = \frac{16 \pm \sqrt{76}}{6} = \frac{16 \pm 2\sqrt{19}}{6} = \frac{8 \pm \sqrt{19}}{3}$$

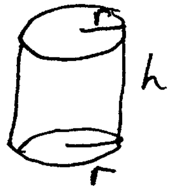
$\frac{8+\sqrt{19}}{3}$ is too big (>3).

x	0	$\frac{8-\sqrt{19}}{3}$	3
V	0	a positive number	0

Answer: max volume when $x = \frac{8-\sqrt{19}}{3}$ inches.

≈ 1.2 in

3.7 #25



Let radius r and height h be measured in inches.

area of top, bottom $= \pi r^2$

area of sides $= 2\pi r h$

Cost $= 2\pi r^2 + 2\pi r^2 + 2\pi r h$ <u>Minimize cost</u>

$= 4\pi r^2 + 2\pi r h.$

Volume $= 12$ fl. oz. $= 21.66$ in^3 (See p. 313)

$\pi r^2 h = 21.66$ so $h = \dfrac{21.66}{\pi r^2}$

$$C = 4\pi r^2 + 2\pi r \frac{21.66}{\pi r^2}$$

$$C = 4\pi r^2 + 2(21.66) r^{-1} \qquad 0 < r < \infty$$

$$C' = 8\pi r - 2(21.66) r^{-2}$$ undefined for $r = 0$.

$$0 = 8\pi r - 2(21.66) r^{-2}$$

$$8\pi r = 2(21.66) r^{-2}$$

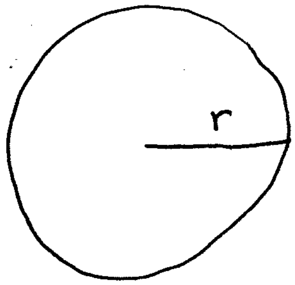
$$r^3 = \frac{2(21.66)}{8\pi} \approx 1.7$$

$r \approx 1.20$ As $r \to 0^+$, $C \to \infty$. As $r \to \infty$, $C \to \infty$

so there must be an absolute minimum at the only critical point in $(0, \infty)$, which is $r \approx 1.20$

$h = \dfrac{21.66}{\pi (1.20)^2} \approx 4.80$ Answer: radius ≈ 1.2
height ≈ 4.8

3.8 # 1a



Let V = volume
r = radius

h = thickness = $\frac{1}{4}''$ (constant).

Know $\frac{dV}{dt} = 120$ gal/min.

Want $\frac{dr}{dt}$ when $r = 100$ ft.

Convert all units to ft.

$$\frac{dV}{dt} = 120 \text{ gal/min} \cdot \frac{1}{7.5} \text{ ft}^3/\text{gal} = 16 \text{ ft}^3/\text{min}$$

$$h = \frac{1}{4} \text{ in} \cdot \frac{1}{12} \frac{\text{ft}}{\text{in}} = \frac{1}{48} \text{ ft}.$$

Equation relating V and r:

$$V = \pi r^2 h$$

$$V = \frac{\pi}{48} r^2$$

$$\frac{dV}{dt} = \frac{\pi}{48} r \frac{dr}{dt}$$ Put in $\frac{dV}{dt} = 16$, $r = 100$

$$16 = \frac{\pi}{48} \cdot 100 \frac{dr}{dt}. \qquad \frac{dr}{dt} = \frac{16 \cdot 48}{\pi \cdot 100} \approx 1.2$$

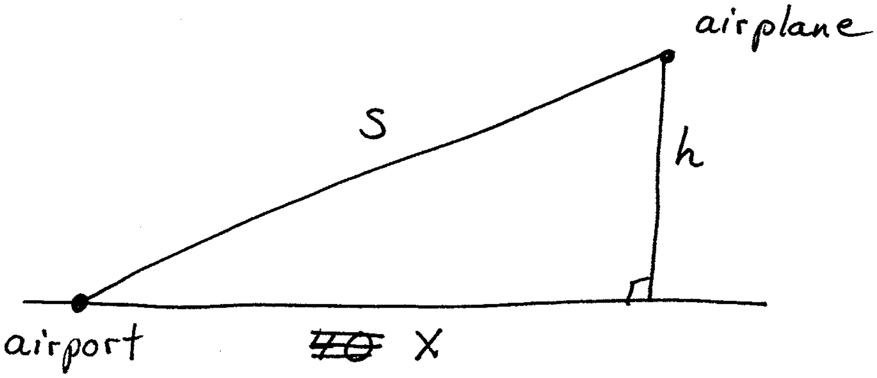
Answer: 1.2 ft/min

(1b) Put in $\frac{dV}{dt} = 16$, $r = 200$ $\qquad 16 = \frac{\pi}{48} \cdot 200 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{16 \cdot 48}{\pi \cdot 200} \approx 0.6$ ft/min ← answer.

When r is smaller, the same change in volume makes a larger change in radius and vice/versa.

3.8 #13



Let x, s, h be as shown, measured in miles.
$h = 4$ (constant). s and x change.

Know $s'(t) = -240$ mi/hr. Find $|x'(t)|$
when $x = 40$ mi.

Equation relating x and s:

$$s^2 = x^2 + h^2$$

$$s^2 = x^2 + 16$$

$$2s\,s' = 2x\,x'.$$

When $x = 40$,
$s^2 = (40)^2 + 16 = 1616$
$s \approx 40.1995$

$$2(40.1995)(-240) = 2 \cdot 40 \cdot x'$$

$$x' = \frac{2(40.1995)(-240)}{2 \cdot 40} = -241.1970$$

Answer: speed is 241.1970 mi/hr.

3.8 #33

Given $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$.

T and ρ are constant and $\sqrt{\frac{T}{\rho}} = 220$ ft/s

so $f = \frac{1}{2L} \cdot 220$

$f = 110\, L^{-1}$. f and L change.

Given $L'(t) = -4$ ft/s, find $f'(t)$.

$f'(t) = -110\, L^{-2}\, L'(t)$

Put in $L'(t) = -4$ and $L = .5$

$f'(t) = -110\, (.5)^{-2}\, (-4) = \boxed{1760 \text{ Hz/sec.}}$

when $L = .5$, $f = 110\, (.5)^{-1} = 220$ Hz

to double this, must increase by 220 Hz

Since $f' = 1760$ Hz/sec, this will take

$$\frac{220 \text{ Hz}}{1760 \text{ Hz/sec}} = \boxed{\frac{1}{8} \text{ sec.}}$$