Math 221 DL1 Test 2 Solutions - white version

(1a) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$. (1b) $\frac{d}{dx} \tan^{-1}(2x) = \frac{2}{1+(2x)^2}$

(2a) $\lim_{x \to \pi} \frac{\sin x}{x^2-\pi^2} = \lim_{x \to \pi} \frac{\cos x}{2x} = \frac{-1}{2\pi}$

→ (2b) $\lim_{x \to \infty} \ln(1+x^{-1})^x = \lim_{x \to \infty} x \ln(1+x^{-1}) = \lim_{x \to \infty} \frac{\ln(1+x^{-1})}{x^{-1}}$

$= \lim_{x \to \infty} \frac{\frac{1}{1+x^{-1}}(-x^{-2})}{-x^{-2}} = \lim_{x \to \infty} \frac{1}{1+x^{-1}} = 1$. $\lim_{x \to \infty} (1+x^{-1})^x = e^1 = e$.

(3a) f has a local min at $x=c$ if $f(c) \le f(x)$ for all x in some open interval containing c.

(3b) x is a critical number for f if $f'(x)=0$ or undefined.

(4) If f is continuous on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ with $f'(c) = \frac{f(b)-f(a)}{b-a}$.

(5) $f'(x) = 1-4x^{-2/3}$. Undefined for $x=0$. $1-4x^{-2/3}=0 \Leftrightarrow$
$1 = 4x^{-2/3} \Leftrightarrow x^{2/3}=4 \Leftrightarrow x = 4^{3/2} = 2^3 = 8$.

x	0	8	-1	27
f(x)	0	-16	11	27-36=-9.

Abs max is 11, at $x=-1$.
Abs. min is -16, at $x=8$

(6) $2x + y + xy' + 2yy' = 0$. when $x=1$, $y=1$,
$2+1+y'+2y'=0$ $y'=-1$. Slope $=-1$

(7) Suppose f has 2 zeroes, a and b. This means $f(a)=f(b)=0$. Since f is continuous and differentiable, we can apply Rolle's Theorem to conclude there is a $c \in (a,b)$ with $f'(c)=0$. But then c is a critical point. Since f has no critical points, it is impossible for f to have 2 zeroes.

→ (8a) increasing on $(3,\infty)$ and $(-1/2, 1/2)$ (8b) concavedown on $\cancel{(0,3)}$ $(0,2)$

(8c) Local min at $x = -1/2$, $x=3$

(9a) T (9b) T (9c) F

Math 221 DL1 Test 2 solutions - ivory version

(1a) $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$ (1b) $\frac{d}{dx}\sin^{-1}\left(\frac{x}{3}\right) = \frac{1}{3}\frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$

→ (2a) $\lim_{x\to\pi}\frac{1+\cos x}{x^2-\pi^2} = \lim_{x\to\pi}\frac{-\sin x}{2x} = \frac{0}{2\pi} = 0$

(2b) $\lim_{x\to\infty}\ln\left(1-\frac{1}{x}\right)^{2x} = \lim_{x\to\infty}2x\ln(1-x^{-1}) = \lim_{x\to\infty}\frac{2\ln(1-x^{-1})}{x^{-1}}$

$= \lim_{x\to\infty}\frac{2\frac{1}{1-x^{-1}}\cdot x^{-2}}{-x^{-2}} = \lim_{x\to\infty}\frac{-2}{1-x^{-1}} = -2.$ $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{2x} = e^{-2} = \frac{1}{e^2}$

(3a) $f(c)$ is an absolute maximum for f if $f(c)\geq f(x)$ for all x (in whatever set is being considered)

(3b) $x=c$ is an inflection point for f if f changes concavity (from up to down or from down to up) at c.

(4) If f is continuous on a closed interval $[a,b]$, then f has an absolute max. and abs. min. on $[a,b]$

(5) $f'(x) = 2x^{-1/3}-1$. Undefined for $\boxed{x=0}$. $2x^{-1/3}-1=0 \Rightarrow$

$2x^{-1/3}=1 \Rightarrow 2 = x^{1/3} \Rightarrow x = 2^3 = 8.$

x	-1	0	1
$f(x)$	4	0	2

The abs max is 4, at $x=-1$.
The abs. min is 0, at $x=0$.

(6) $3x^2-2y-2xy'+2yy'=0$. When $(x,y)=(0,2)$,
$0-4-0+4y'=0$. $y'=1$. Slope $=1$.

(7) See #7 on the white version solutions. It is the same question.

(8a) decreasing on $(-\infty,\frac{-1}{2})$ and on $(\frac{1}{2},3)$

→ (8b) concave up on $(-\infty,0)$ and on ~~$(\cancel{x},8)$~~ ~~$(2,8)$~~ $(2,\infty)$

(8c) Local max. at $x=\frac{1}{2}$.

(9a) F (9b) T (9c) T