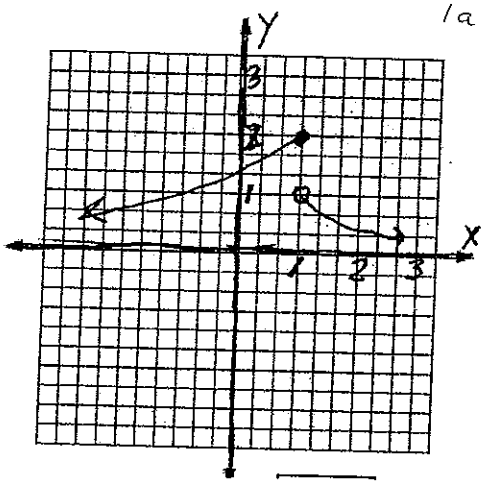
1. (7 points each part) Evaluate each limit as a number, as ∞ or $-\infty$, or say "does not exist." Show your work or give a brief explanation in words as to how you arrived at your answer. These limits should be done without the use of L'Hopital's Rule, which we have not covered yet.

(a)



$\lim_{x \to 1} f(x)$ does not exist

because $\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1^-} f(x) = 2$

(b)

$$\lim_{x \to \infty} \frac{2x^2 - 1}{4x^3 - x} =$$

$$\lim_{x \to \infty} \frac{\frac{2x^2}{x^3} - \frac{1}{x^3}}{\frac{4x^3}{x^3} - \frac{x}{x^3}} = \lim_{x \to \infty} \frac{\frac{2}{x} - \frac{1}{x^3}}{4 - 1/x^2} = \frac{0}{4} = 0$$

(c)

$$\lim_{x \to 0} \frac{\tan x}{x}$$

$$= \lim_{x \to 0} \frac{\sin x / \cos x}{x} = \lim_{x \to 0} \frac{1}{\cos x} \cdot \frac{\sin x}{x} = 1 \cdot 1 = 1$$

2. (8 points each part) Find each derivative. You can use any of the derivative rules we have studied.

(a)

$$\frac{d}{dx}\sqrt{x^4+x^2}$$

$$= \frac{d}{dx}(x^4+x^2)^{1/2} = \frac{1}{2}(x^4+x^2)^{-1/2}(4x^3+2x)$$

chain rule

(b)

$$\frac{d}{dx}e^x \tan x$$

$$= e^x \frac{d}{dx}\tan x + \frac{d}{dx}(e^x)\tan x$$

$$= e^x \sec^2 x + e^x \tan x$$ product rule

(c)

$$\frac{d}{dx}(3x)^{\sin x}$$

logarithmic differentiation

$y = (3x)^{\sin x}$

$\ln y = \ln (3x)^{\sin x}$

$\ln y = (\sin x) \ln(3x)$

$\frac{1}{y} \frac{dy}{dx} = (\cos x) \ln(3x) + (\sin x) \frac{1}{3x} \cdot 3$

$\frac{dy}{dx} = y\left[\cos x \ln(3x) + \frac{1}{x} \sin x\right]$

$\frac{dy}{dx} = (3x)^{\sin x}\left[\cos x \ln(3x) + \frac{1}{x} \sin x\right]$

(d) $\frac{dy}{dx}$ at $x = 1$, given that $y = f(g(x))$ and
$g(1) = -4$,
$f(1) = 7$,
$g'(1) = 5$,
~~$g'(1) = 3$,~~
$g'(-4) = 2$,
$f'(-4) = 6$,
$g'(7) = 1$,
$f'(7) = -8$.

chain rule.

$\frac{dy}{dx} = f'(g(x))\, g'(x)$

$= f'(g(1))\, g'(1)$

$= f'(-4)\, g'(1)$

$= 6 \cdot 5$

$= 30$

3. (a) (5 points) State the definition of the derivative $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

(b) (8 points) Use the definition of the derivative to show that $f'(2) = -1/4$ for $f(x) = \frac{1}{x}$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2-(2+h)}{(2+h)2}}{h} = \lim_{h \to 0} \frac{\frac{-h}{(2+h)(2)}}{h}$$

$$= \lim_{h \to 0} \frac{-1}{(2+h)(2)} = \frac{-1}{2 \cdot 2} = \frac{-1}{4}$$

4. (10 points) State the Squeeze Theorem.

If $f(x) \le g(x) \le h(x)$ for all x near $x = a$

and if $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$

then $\lim_{x \to a} g(x) = L$.

5. (6 points each part) Give an example of each of the following. Your example can be given as a formula or as a graph, as long as the graph unambiguously shows the relevant features of the example.

(a) Give an example of a function $f(x)$ which is continuous at $x = 0$ but not differentiable at $x = 0$.

$f(x) = |x|$ is one example.

Also $f(x) = x^{1/3}$

(b) Give an example of a function $f(x)$ for which $f''(x) = 0$ for all x.

$f(x) = x$ is one example.

(any function whose graph is a straight line has $f''(x) = 0$)

6. (4 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If f is discontinuous at $x = 0$ and if

$$\lim_{x \to 0} f(x) = 1,$$

then the discontinuity at $x = 0$ is removable.

T (the fact that $\lim_{x \to 0} f(x)$ exists is the key fact).

(b) If $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} g(x) = 0$, then

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

does not exist.

F Here's an example: $\lim_{x \to \infty} \frac{x^{-1}}{x^{-1}} = 1$

(c) Polynomials are continuous at all values of x.

T See page 100.