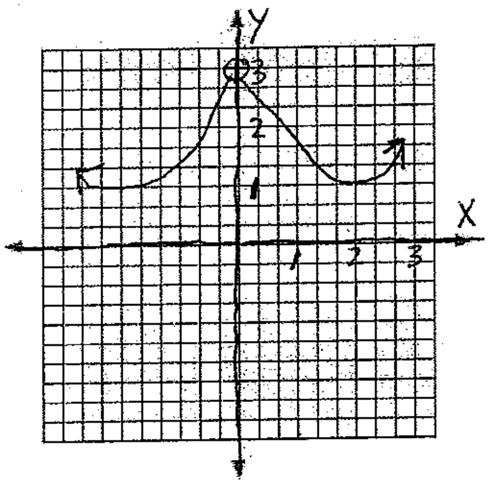
1. (7 points each part) Evaluate each limit as a number, as ∞ or $-\infty$, or say "does not exist." Show your work or give a brief explanation in words as to how you arrived at your answer. These limits should be done without the use of L'Hopital's Rule, which we have not covered yet.

(a)



$\lim_{x\to 0} f(x) = 3$. From the graph, we see that as x approaches 0 from the right or the left, y approaches 3.

(b)

$$\lim_{x\to 3^-} \frac{1}{\sqrt{9-x^2}} = \infty$$

As x approaches 3 from the left, $\sqrt{9-x^2}$ approaches 0 but is always positive, so the limit is $+\infty$.

(c)

$$\lim_{x\to 0} x \csc x = 1$$

$$= \lim_{x\to 0} x \cdot \frac{1}{\sin x} = \lim_{x\to 0} \frac{x}{\sin x} = \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}$$

$$= \frac{1}{1} = 1.$$

2. (8 points each part) Find each derivative. You can use any of the derivative rules we have studied.

(a)

$$\frac{d}{dx}\sqrt[5]{7x - x^3}$$

$$= \frac{d}{dx}(7x - x^3)^{1/5} = \frac{1}{5}(7x - x^3)^{-4/5}(7 - 3x^2)$$

chain rule.

(b)

$$\frac{d}{dx}(\ln x)(\sec x)$$

$$= \frac{1}{x}\sec x + (\ln x)\sec x \tan x$$

(product rule)

(c)

$$\frac{d}{dx}(\cos x)^x$$

$y = (\cos x)^x$

$\ln y = \ln (\cos x)^x$

$\ln y = x \ln(\cos x)$

$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln(\cos x) + x \frac{1}{\cos x}(-\sin x)$

$= \ln(\cos x) - \frac{x \sin x}{\cos x}$

$\frac{dy}{dx} = y\left[\ln(\cos x) - \frac{x\sin x}{\cos x}\right]$

$= (\cos x)^x\left[\ln(\cos x) - \frac{x \sin x}{\cos x}\right]$

(d) $\frac{dy}{dx}$ at $x = 0$, given that $y = f(g(x))$ and
$g(0) = -1$,
$f(0) = 2$,
$g'(0) = 3$,
$g'(-1) = -5$,
$g'(2) = 8$,
$f'(0) = 4$,
$f'(-1) = 7$,
$f'(2) = 6$.

$\frac{dy}{dx} = f'(g(x))\, g'(x)$ chain rule

at $x = 0$,

$\frac{dy}{dx} = f'(g(0))\, g'(0)$

$= f'(-1)\, g'(0)$

$= 7 \cdot 3$

$= 21$

3. (a) (5 points) State the definition of the derivative $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(b) (8 points) Use the definition of the derivative to show that $f'(4) = 1/4$ for $f(x) = \sqrt{x}$.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{4+h} - 2)}{h} \frac{(\sqrt{4+h} + 2)}{(\sqrt{4+h} + 2)}$$

$$= \lim_{h \to 0} \frac{4+h-4}{h(\sqrt{4+h} + 2)} = \lim_{h \to 0} \frac{h}{h(\sqrt{4+h} + 2)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

4. (10 points) State the Squeeze Theorem.

If $f(x) \le g(x) \le h(x)$ for all x near a (but not necessarily at $x=a$) and if

$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$,

then $\lim_{x \to a} g(x) = L$

5. (6 points each part) Give an example of each of the following. Your example can be given as a formula or as a graph, as long as the graph unambiguously shows the relevant features of the example.

(a) Give an example of a function $f(x)$ which has discontinuities at $x = -1$ and at $x = 1$ and is continuous at all other values of x.

$f(x) = \frac{1}{(x+1)(x-1)}$ is one example.

(b) Give an example of a function $f(x)$ for which $f''(x) > 0$ for all x.

$f(x) = x^2$ is one example:

$f'(x) = 2x$

$f''(x) = 2 \leftarrow$ greater than 0 for all x.

6. (4 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If f is discontinuous at $x = 0$ and if

$$\lim_{x \to 0} f(x) = 1,$$

then the discontinuity at $x = 0$ is nonremovable.

F It is removable by defining $f(0) = 1$.

(b) Polynomials are continuous at all values of x.

T See page 100

(c) If $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} g(x) = 0$, then

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

does not exist.

F. Here's an example:

$$\lim_{x \to \infty} \frac{x^{-2}}{x^{-1}} = \lim_{x \to \infty} x^{-1} = 0.$$