

Exam 3 Information and Review - Math 221, Sections AL1, DL1, Fall 2007

- This exam will cover sections 3.8, 4.1-4.6, 5.1-5.3.
- Monday, December 3, 314 Altgeld Hall, 9:00-9:50 for Section AL1, 1:00-1:50 for Section DL1
- **Please bring your I-card to the exam.**
- No calculators, books, or notes allowed. You can not use any headphones during the exam.
- Seating will be assigned when you enter the room.
- Left-handed people: you may take the test in
341 Altgeld Hall (for Section AL1, 9:00)
143 Altgeld Hall (for Section DL1, 1:00).
The room can hold about 30 people - first come, first served!

The review problems below represent kinds of problems that may appear on the exam. They do *not* include every possible topic that may be covered on the exam. Also, the exam will be quite a bit shorter than this review page! Re-doing homework problems and examples without looking at your book or notes is also good practice.

1. Related Rates. A spherical balloon is inflated at a rate of 10 cubic inches per minute. How fast is the radius of the balloon increasing at the moment when the radius is 5 inches? You are given the following formula for the volume V of a sphere in terms of its radius r .

$$V = \frac{4}{3}\pi r^3$$

2. Related Rates. Maple and Main Streets are straight and perpendicular to each other. A police car is parked on Main Street $1/4$ mile from the intersection of the two streets. A motorcycle on Maple Street approaches the intersection at 40 miles per hour. How fast is the distance between the police car and the motorcycle decreasing when the motorcycle is $1/8$ mile from the intersection?
3. Related Rates. The base of a rectangle is increasing at 4 cm/sec while its height is decreasing at 3 cm/sec. At what rate is its area changing when its base is 20 cm and its height is 12 cm?
4. Evaluate each of the following indefinite integrals.

(a)

$$\int \sqrt{x} \, dx$$

(b)

$$\int \csc^2 \theta \, d\theta$$

(c)

$$\int x^{-1} \, dx$$

(d)

$$\int 2e^{-t} dt$$

(e)

$$\int x\sqrt[3]{1-x^2} dx$$

(f)

$$\int \frac{1}{x^2+1} dx$$

(g)

$$\int \frac{x}{x^2+1} dx$$

(h)

$$\int \frac{x}{x^4+1} dx$$

(i)

$$\int \frac{x^4+1}{\sqrt{x}} dx$$

(j)

$$\int 3\cos x + 4\sin x dx$$

5. Compute the sum by writing out the terms and adding. The point of this exercise is to check that you understand the Σ notation.

$$\sum_{i=2}^4 (2^i + 3i)$$

6. Use summation formulas to evaluate. The summation formulas can be found in Theorem 2.1 on page 356 and these will be given to you on Exam #3 and on the final exam.)

$$\sum_{i=1}^{100} (4 + 3i - i^2)$$

7. Compute. You may use the summation formulas mentioned in the previous problem.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[\left(\frac{i}{n} \right)^2 - 5 \left(\frac{i}{n} \right) \right]$$

BONUS: What definite integral is represented by the above limit? Evaluate the definite integral and check that you get the same number.

8. Problem 1b from Section 4.3

9. Problem 3a from Section 4.3

10. Be able to give the definition of the definite integral, explaining what all the notation means. Note: this is the major definition of Chapter 4. It is almost certain that on Exam 3 and/or the final, you will be asked to state this definition and to evaluate some definite integral using the definition.

11. Use the definition of definite integral to evaluate the following. Of course you can use the Fundamental Theorem of Calculus to check your answer.

$$\int_1^3 (x^2 - x) dx$$

12. Sketch the graph of $y = |x| + 1$ and use geometric formulas to evaluate

$$\int_{-1}^2 (|x| + 1) dx.$$

13. Find the average value of $f(x) = \cos(2x)$ on the interval $[0, \pi/2]$.

14. Find $f'(2)$ for

$$f(x) = \int_0^{x^2} t 3^t dt.$$

15. Find the area bounded by the $y = x + 2$, $y = -3x + 6$, $y = 1$.

16. Find the area bounded by $x = y^2$ and $x = y + 6$.

17. As viewed from above, a swimming pool has the shape of a circle

$$x^2 + y^2 = 900.$$

The cross sections of the pool perpendicular to the ground are squares. If the units are feet, determine the volume of the pool.

18. Section 5.2, problem 26. Any of the parts are good practice.

19. Section 5.3, problem 1.

20. Section 5.3, problem 3.

21. Answer true or false for each of the following.

(a) Every function has an antiderivative.

(b) If $f(x)$ is continuous on $[a, b]$, then $f(x)$ has an antiderivative on $[a, b]$.

(c) If $f(x)$ is continuous on $[a, b]$ and if $f(x) \geq 0$ on $[a, b]$, then $\int_a^b f(x) dx \geq 0$.

(d)

$$\int_{-1}^1 \frac{1}{x^2} = -2.$$

- (e) The Mean Value Theorem is used in proving the Fundamental Theorem of Calculus.
- (f) If $f(x)$ is continuous on $[2, 7]$ and if $\int_2^7 f(x) dx = 15$, then there is a number c between 2 and 7 with $f(c) = 3$.
- (g) If f is integrable on $[a, b]$, then f is differentiable on (a, b) .
- (h) If f is continuous for all x , then

$$\int_0^{10} f(x) dx + \int_{10}^5 f(x) dx = \int_0^5 f(x) dx.$$

- (i) $\int_1^1 f(x) dx = 0$.
- (j) For an integrable function $f(x)$,

$$\int_a^b f(x) dx = \frac{1}{2} \int_a^b 2f(x) dx.$$

- (k) For an integrable function $f(x)$,

$$\int_a^b f(x) dx = \frac{1}{x} \int_a^b xf(x) dx.$$

- (l) If f and g are both integrable, then

$$\int_a^b f(x)g(x) dx = \left(\int_a^b f(x) dx \right) \left(\int_a^b g(x) dx \right)$$

22. State the theorem. Your answer should clearly indicate what the hypotheses and conclusion are, for example by using "if...then".

- (a) Fundamental Theorem of Calculus, Part I
- (b) Fundamental Theorem of Calculus, Part II