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Math 221, AL1 - Test #2 - October 22, 2007

Name:____

Signature:____

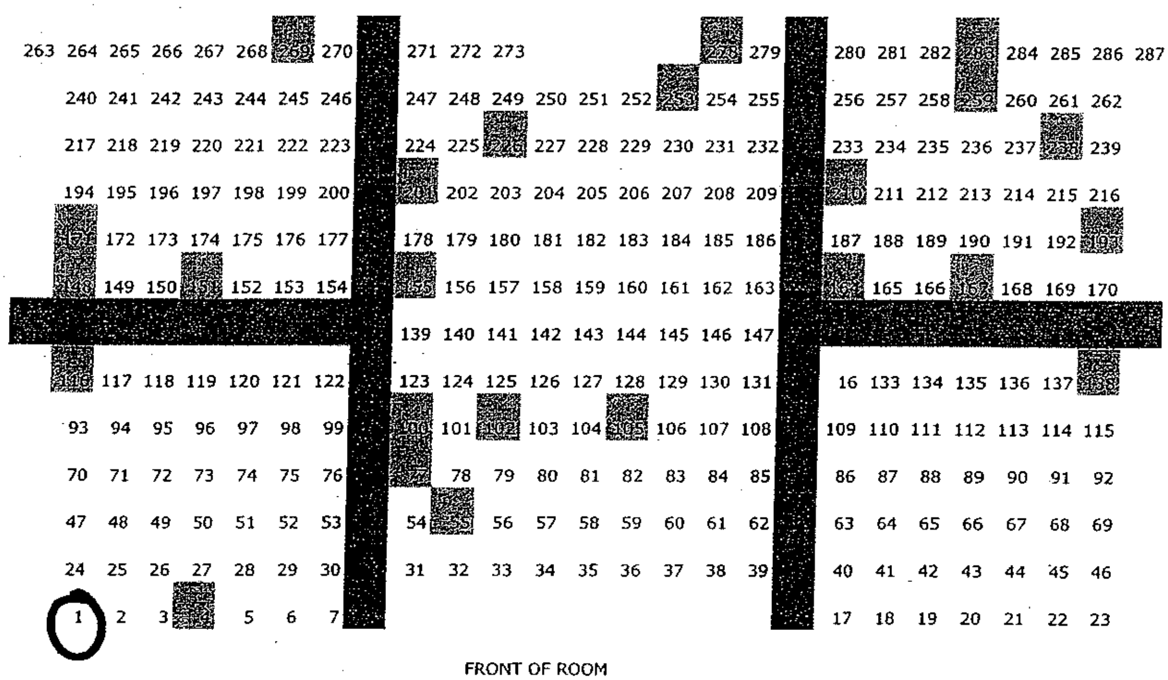
Circle your Section: AD1 (8:00,Wojciech Samotij) AD2 (9:00,Tim LeSaulnier)

AD3 (1:00,Kunwoo Kim) AD4 (3:00,Chris Appuhn) AD5 (10:00,Wojciech Samotij)

AD6 (1:00,Patricia LeVon) AD7 (12:00,Kunwoo Kim) AD8 (2:00,Chris Appuhn)

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

SIT IN THE SEAT CIRCLED BELOW.



Time: 50 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.

problem number	1	2	3	4	5	6	7	8
possible points	7	10	10	16	20	15	10	12
score								

1. (7 points) Find $f'(x)$ for

$$f(x) = \sin^{-1}(2x).$$

$$f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

2. (10 points) On which intervals is the graph of $f(x) = x^3 - 6x^2 + 3x - 1$ concave up and on which intervals is it concave down?

$$f'(x) = 3x^2 - 12x + 3$$

$$f''(x) = 6x - 12$$

$f''(x) = 0$ for $x = 2$

$-$ $\quad$ $+$ $\quad$ f''

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concave down on $(-\infty, 2)$

concave up on $(2, \infty)$

3. (10 points) Use implicit differentiation to find dy/dx for

$$x^5 + y^5 = 5x^2y^2.$$

$$5x^4 + 5y^4 \frac{dy}{dx} = 10xy^2 + 10x^2y \frac{dy}{dx}$$

$$\left(5y^4 - 10x^2y\right)\frac{dy}{dx} = 10xy^2 - 5x^4$$

$$\frac{dy}{dx} = \frac{10xy^2 - 5x^4}{5y^4 - 10x^2y}$$

4. (8 points each part) Evaluate each limit.

(a)

$$\lim_{x\to 0} \frac{e^{3x}-1}{x}$$

type $\frac{0}{0}$

$$= \lim_{x\to 0} \frac{3e^{3x}}{1} = \frac{3e^0}{1} = 3$$

(b)

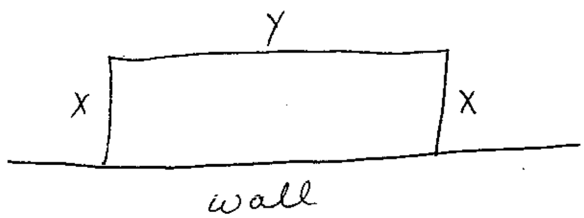
$$\lim_{x\to 1}(x-1))^{-1}\ln x$$

type $\infty \cdot 0$

$$= \lim_{x\to 1} \frac{\ln x}{x-1}$$ type $\frac{0}{0}$

$$= \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \frac{1}{1} = 1$$

5. (a) (15 points) A farmer has 400 ft. of fencing with which to enclose a rectangular pen adjacent to an existing wall. He will use the wall for one side of the pen and the fencing for the other 3 sides. He wants to make the area of the pen as large as possible. What is the maximum area of such a pen?



Let x, y be as shown, in feet.

Maximize area $= A = xy$

Since $2x + y = 400$, $y = 400 - 2x$

$A = x(400 - 2x) = 400x - 2x^2$

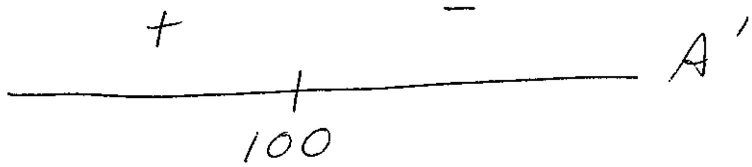
$A' = 400 - 4x$

$A' = 0$ for $x = 100$

$y = 400 - 2(100) = 200$

area $= 100 \cdot 200 = 20000$ sq. ft.

(b) (5 points) Explain mathematically how you know that the area you found in part (a) is the maximum possible.



Since $A' > 0$ for $x < 100$ and $A' < 0$ for $x > 100$,

A has a maximum at $x = 100$.

6. (a) (5 points) Write the formula which is used in Newton's method, giving x_{n+1} in terms of x_n. You do not need to show where the formula comes from.

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

(b) (10 points) Use Newton's method to estimate $\sqrt{8}$. Carry out two steps of Newton's method. You do not need to simplify the result of the second step.

Let $f(x) = x^2 - 8$

$f'(x) = 2x$

Choose $X_0 = 3$ (close to $\sqrt{8}$)

$$X_1 = 3 - \frac{9-8}{6} = \frac{18}{6} - \frac{1}{6} = \frac{17}{6}$$

$$X_2 = \frac{17}{6} - \frac{\left(\frac{17}{6}\right)^2 - 8}{2 \cdot \frac{17}{6}}$$

7. (10 points) State the Mean Value Theorem.

If f is continuous on $[a, b]$
and differentiable on (a, b)

then there is a number c between
a and b with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

8. (4 points each part) Answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If $f(x)$ does not have an absolute maximum on a closed interval $[a, b]$, then $f(x)$ is not continuous on $[a, b]$.

T (If f were cont., it would have abs. max. and min. by Extreme Value Theorem)

(b) If $f'(3) = 0$ and if $f''(3) = 1$, then $f(x)$ has a local maximum at $x = 3$.

$\cup$ F f has a local min. at $x = 3$ by the 2^{nd} derivative test.

(c) The linear approximation to $f(x)$ at $x = x_0$ is usually a poor approximation to $f(x)$ when x is very close to x_0.

F   It is usually a good approx. when x is close to x_0.