

Math 221, AL1 - Test #2 - October 22, 2007

Name: _____

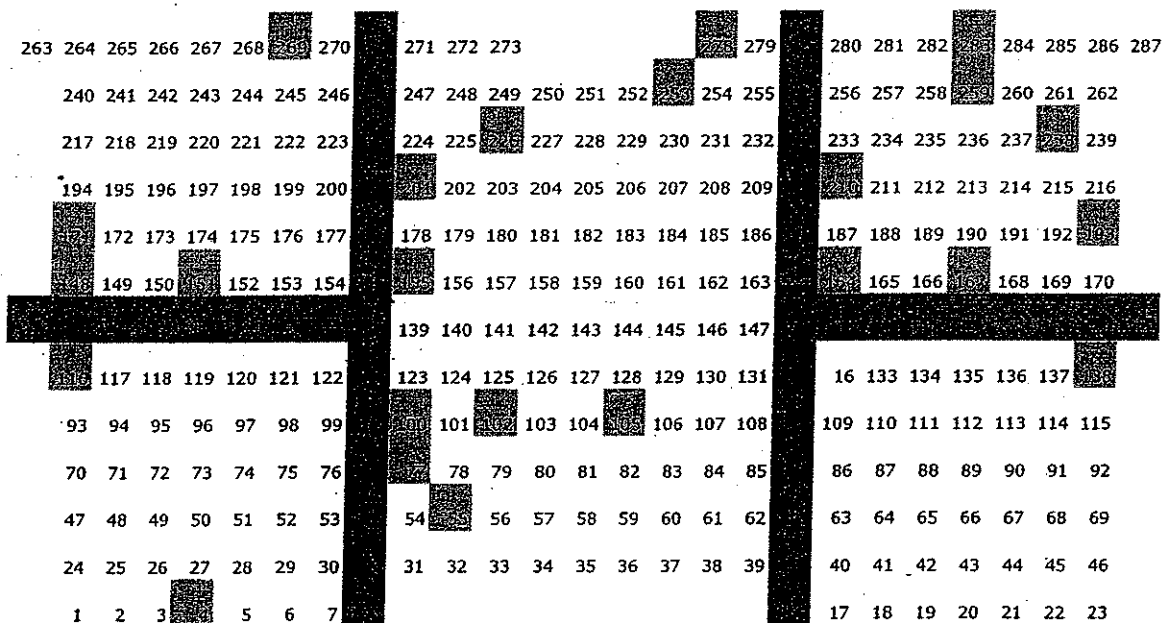
Signature: _____

Circle your Section: AD1 (8:00,Wojciech Samotij) AD2 (9:00,Tim LeSaulnier)

AD3 (1:00,Kunwoo Kim) AD4 (3:00,Chris Appuhn) AD5 (10:00,Wojciech Samotij)

AD6 (1:00,Patricia LeVon) AD7 (12:00,Kunwoo Kim) AD8 (2:00,Chris Appuhn)

DO NOT OPEN EXAM UNTIL TOLD TO DO SO SIT IN THE SEAT CIRCLED BELOW.



FRONT OF ROOM

Time: 50 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.

problem number	1	2	3	4	5	6	7	8
possible points	7	10	10	16	20	15	10	12
score								

1. (7 points) Find $f'(x)$ for

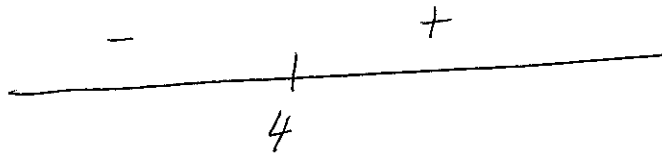
$$f(x) = \tan^{-1}(x^3).$$

$$f'(x) = \frac{1}{1 + (x^3)^2} \cdot 3x^2 = \frac{3x^2}{1 + x^6}$$

2. (10 points) On which intervals is the graph of $f(x) = x^2 - 8x + 11$ increasing and on which intervals is it decreasing?

$$f'(x) = 2x - 8$$

$$f'(x) = 0 \quad \text{for} \quad x = 4$$



increasing on $(4, \infty)$

decreasing on $(-\infty, 4)$.

3. (10 points) Use implicit differentiation to find dy/dx for

$$x^2 - x^2y = y^2x + y^3.$$

$$2x - 2xy - x^2 \frac{dy}{dx} = 2yx \frac{dy}{dx} + y^2 + 3y^2 \frac{dy}{dx}$$

$$2x - 2xy - y^2 = (2xy + 3y^2 + x^2) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xy + 3y^2 + x^2}{2x - 2xy - y^2}$$

4. (8 points each part) Evaluate each limit.

(a)

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x}$$

type $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$$

(b)

$$\lim_{x \rightarrow 0} x^{-1} \sin(2x)$$

type $(\pm\infty) \cdot 0$

$$= \lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$$

type $\frac{0}{0}$

$$= \lim_{x \rightarrow 0} \frac{2 \cos(2x)}{1} = \frac{2 \cos(0)}{1} = 2$$

6. (a) (5 points) Give the formula for the linear approximation of $f(x)$ at $x = x_0$. Your answer should be a function of x . You do not need to explain where your answer comes from.

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

- (b) (10 points) Use linear approximation to estimate $\sin(0.1)$. Be sure to indicate what you are using for $f(x)$ and for x_0 .

$$\text{Let } f(x) = \sin x$$

$$x_0 = 0$$

$$x = 1$$

$$f'(x) = \cos x$$

$$f(x_0) = 0, \quad f'(x_0) = 1$$

$$L(1) = 0 + 1(1 - 0) = 1$$

7. (10 points) State the Extreme Value Theorem.

If f is continuous on $[a, b]$,
then f has an absolute max.
and an absolute min. on $[a, b]$.

8. (4 points each part) Answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If $f(x)$ is continuous and differentiable for all x and if $f(2) = 2$ and $f(3) = 5$, then there must be a number c with $f'(c) = 0$.

F. The Mean Value Theorem guarantees
there is a c with $f'(c) = \frac{5-2}{3-2} = 3$.

(b) If $x = c$ is a critical number for $f(x)$ then $f(x)$ has either a local maximum or a local minimum at $x = c$.

F. $f(x) = x^3$, $c = 0$ is a counterexample.

(c) Newton's method is used to approximate a zero of a function.

T