

Exam 2 Information and Review - Math 221, Sections AL1, DL1, Fall 2007

- This exam will cover sections 2.8, 2.9, 3.1-3.5, 3.7.
- Monday, October 22, 314 Altgeld Hall, 9:00-9:50 for Section AL1, 1:00-1:50 for Section DL1
- **Please bring your I-card to the exam.**
- No calculators, books, or notes allowed. You can not use any headphones during the exam.
- Seating will be assigned when you enter the room.
- Left-handed people: you may take the test in
341 Altgeld Hall (for Section AL1, 9:00)
143 Altgeld Hall (for Section DL1, 1:00).
The room can hold about 30 people - first come, first served!

The review problems below represent kinds of problems that may appear on the exam. They do *not* include every possible topic that may be covered on the exam. Also, the exam will be quite a bit shorter than this review page! Re-doing homework problems without looking at your book or notes is also good practice. Section DL1 - please do not neglect the homework problems which were to be done on paper.

1. For this test (and for the rest of this course), you should know the derivatives of $\sin^{-1} x$ and of $\tan^{-1} x$. These are the two inverse trig functions which are most frequently used. Know where to look up the derivatives of the other inverse trig functions in case you need them for a homework problem or for another class (page 222).

2. For each of the following, find the indicated derivative.

- (a) Find dy/dx , also known as $y'(x)$, for

$$xe^y - 3y \sin x = 1.$$

- (b) Find $f'(x)$ for

$$f(x) = \tan^{-1}(x^2 - x).$$

- (c) Find $f'(x)$ for

$$f(x) = e^{\sin^{-1} x}.$$

3. State the Mean Value Theorem.
4. State the Extreme Value Theorem.
5. Find the linear approximation to $f(x) = (x + 1)^{1/3}$ at $x_0 = 0$.
6. Outline how you would use linear approximations to estimate $\sqrt[3]{28}$. Be as specific as possible without carrying out any calculations which require a calculator.
7. Outline how you would use Newton's method to estimate $\sqrt[3]{28}$. Carry out as many steps as are practical without a calculator.

8. Give the statement of L'Hopital's rule. Be sure you know the conditions under which it can be applied! (the primary hypothesis is that the limit be of indeterminate form $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.)
9. Evaluate each of the following limits. If you use L'Hopital's rule, be sure that you have an indeterminate form of type $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$. (on the test, you'll need to remember this.)

(a)

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$$

(c)

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x^{-1} - 1}$$

(d)

$$\lim_{x \rightarrow \infty} x e^{-x}$$

(e)

$$\lim_{x \rightarrow 3} e^x x^{-1}$$

(f) (this one is hard!)

$$\lim_{x \rightarrow 3^+} (x - 2)^{\ln(x-3)}$$

(g)

$$\lim_{x \rightarrow 0} \frac{1}{x^3} - \frac{1}{x^4}$$

10. Find all local maxima and local minima for $f(x) = 2x\sqrt{x+1}$

11. Find all local maxima and local minima for

$$f(x) = (x^{2/5} - 3x^{1/5})^2$$

12. Find the absolute maximum and absolute minimum of $f(x) = \sin x \cos x$ on the interval $[0, 2\pi]$. Also give the intervals on which the $f(x)$ is increasing and decreasing.

13. Find all local maxima, local minima and inflection points for $f(x) = e^{-x} \sin x$.

14. Suppose you want to fence in a rectangular corral with an area of 1200 square feet. The front side will be made from fencing which costs \$5 per foot. The other three sides are made from cheaper fencing which costs \$3 per foot. Find the dimensions of the corral which will give you the lowest cost. Also find how much this fence will cost in total.

15. Examples 7.1 and 7.2 and Exercises 3, 4, 5, 6, 15, 16 from Section 3.7 are also recommended practice.

16. For each part, answer true (T) or false (F). No explanation is needed and there is no partial credit.

(a)

$$\sin^{-1} x = \frac{1}{\sin x}.$$

(b)

$$\sin(\sin^{-1}(\theta)) = \theta.$$

(c) If $f'(x) = 0$ for all x , then $f(x)$ is a constant function.

(d) If we iterate Newton's method enough times, it will eventually give us the exact value of a zero of the function $f(x)$.

(e) No matter what initial guess x_0 we begin with, successive iterations of Newton's method always get closer and closer to a zero of the function $f(x)$.

(f) L'Hopital's Rule can be applied whenever we're finding the limit of a quotient.

(g) If $f(x)$ is a polynomial, then

$$\lim_{x \rightarrow \infty} \frac{f(x)}{e^x} = 0.$$

(h) If $f(x)$ is a polynomial, then

$$\lim_{x \rightarrow 0} \frac{f(x)}{e^x} = 0.$$

(i)

$$\lim_{x \rightarrow \infty} x^x$$

has an indeterminate form.

(j)

$$\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^x$$

has an indeterminate form.

(k) If a limit has an indeterminate form, then the limit does not exist.

(l) If a limit has an indeterminate form, then it is a good idea to try to use L'Hopital's Rule.

(m) If $f(x)$ is a function on a closed (and bounded) interval $[a, b]$, then $f(x)$ has an absolute maximum and an absolute minimum on $[a, b]$.

(n) It is possible for the absolute minimum of a function to occur at more than one x -value.

(o) If $x = c$ is a critical number for $f(x)$, then $f(x)$ has a local maximum or a local minimum at $x = c$.

(p) If $f(x)$ has a local maximum or a local minimum at $x = c$, then $x = c$ is a critical number for $f(x)$.

(q) The absolute maximum of a continuous function $f(x)$ on a closed interval $[a, b]$ must occur either at a critical point or at an endpoint of the interval.

(r) If $f'(0) = 0$, $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then $f(x)$ has a local maximum at $x = 0$.

(s) If $f''(c) = 0$, then $f(x)$ has an inflection point at $x = c$.

- (t) If $f''(x) > 0$ on an interval, then the graph of $f(x)$ is concave up on that interval.
- (u) If $f'(3) = 0$ and $f''(3) = 1$, then $f(x)$ has a local maximum at $x = 3$.
- (v) If $f'(-1) = 0$ and $f''(-1) = 0$, then $f(x)$ has neither a local maximum nor a local minimum at $x = -1$.
- (w) If $f''(x) > 0$ for all x , then $f(x)$ is an increasing function.

17. Give an example of each of the following. The example can be given as a formula or as a graph as long as the relevant features of the graph are very clearly drawn so there is no ambiguity about what is meant:

- (a) A function which is continuous on the interval $(0, 5)$ and which has an absolute maximum but no absolute minimum on $(0, 5)$.
- (b) A function which has a local minimum and a local maximum on $(-\infty, \infty)$, but no absolute minimum.
- (c) A function which is concave up and increasing on $(-\infty, \infty)$.
- (d) A function which is concave up and decreasing on $(-\infty, \infty)$.
- (e) A function which is concave down and increasing on $(-\infty, \infty)$.
- (f) A function which is concave down and decreasing on $(-\infty, \infty)$.
- (g) See exercises 47-51 in Section 3.3, exercises 35-38 in Section 3.4 and exercises 41-44 in Section 3.5 for more practice.

18. Know how to get the formula

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

using implicit differentiation.

19. Be able to give the definition of

- (a) the linear approximation of $f(x)$ at $x = x_0$.
- (b) absolute maximum, absolute minimum
- (c) local maximum, local minimum
- (d) critical number (also called critical point)
- (e) increasing on an interval, decreasing on an interval
- (f) concave up, concave down
- (g) inflection point