

Math 221, DL1 - Test #2 - October 22, 2007

Name: _____

Signature: _____

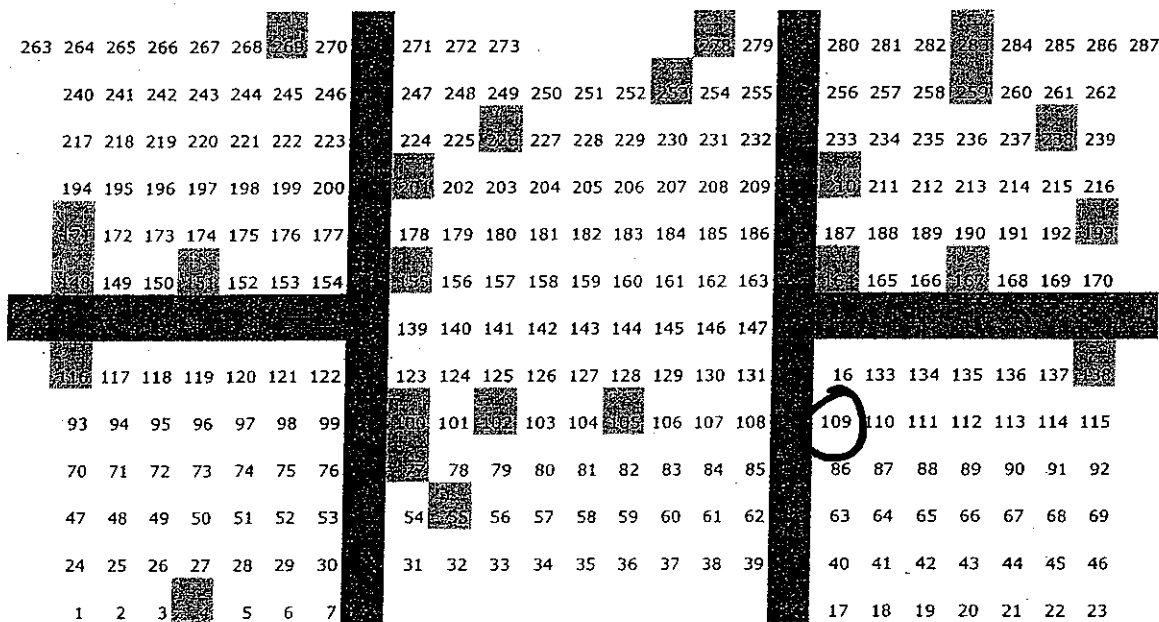
Circle your Section: DD1 (8:00, Aaron Ziegler) DD2 (9:00, John Maki)

DD3 (3:00, Aaron Ziegler) DD4 (1:00, Kevin Milans) DD5 (12:00 Kevin Milans)

DD6 (2:00, Suil O) DD7 (10:00, Suil O) DD8 (12:00, John Lenz)

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DO NOT OPEN EXAM UNTIL TOLD TO DO SO SIT IN THE SEAT CIRCLED BELOW.



FRONT OF ROOM

Time: 50 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.

problem number	1	2	3	4	5	6	7	8
possible points	7	10	10	16	20	15	10	12
score								

1. (7 points) Find $f'(x)$ for

$$f(x) = \sin^{-1}(x^2).$$

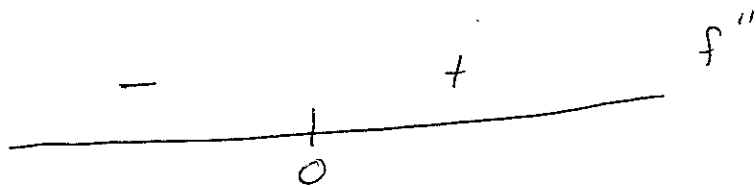
$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

2. (10 points) On which intervals is the graph of $f(x) = 2x^3 - 24x + 1$ concave up and on which intervals is it concave down?

$$f'(x) = 6x^2 - 24$$

$$f''(x) = 12x$$

$$f''(x) = 0 \text{ for } x = 0$$



concave down on $(-\infty, 0)$.

concave up on $(0, \infty)$.

3. (10 points) Use implicit differentiation to find dy/dx for

$$x^4 + x^2y^2 + y^4 = 48.$$

$$4x^3 + 2xy^2 + 2x^2y \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 0$$

$$(2x^2y + 4y^3) \frac{dy}{dx} = -(4x^3 + 2xy^2)$$

$$\frac{dy}{dx} = \frac{-(4x^3 + 2xy^2)}{2x^2y + 4y^3}$$

4. (8 points each part) Evaluate each limit.

(a)

$$\lim_{x \rightarrow 0} \frac{\sin x}{e^x - 1}$$

type $\frac{0}{0}$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{e^x} = \frac{\cos 0}{e^0} = \frac{1}{1} = 1$$

(b)

$$\lim_{x \rightarrow \infty} x^{-1} \ln x$$

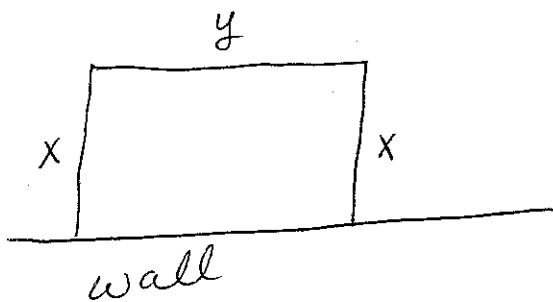
type $0 \cdot \infty$

$$= \lim_{x \rightarrow \infty} \frac{\ln x}{x}$$

type $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0.$$

5. (a) (15 points) A farmer wants to fence in a rectangular pen against a long wall, with the wall forming one side of the pen and the other three sides made out of fencing. The area of the pen is to be 50 square feet. What is the minimum total length of fencing with which such a pen can be made?



Let x, y be as shown, in feet.

Minimize length $L = 2x + y$.

Since $xy = 50$, $y = 50x^{-1}$ and

$$L = 2x + 50x^{-1}$$

$$L' = 2 - 50x^{-2}$$

$$2 - 50x^{-2} = 0$$

$$2 = 50x^{-2}$$

$$25 = x^2$$

$$x = 5 \text{ ft.}$$

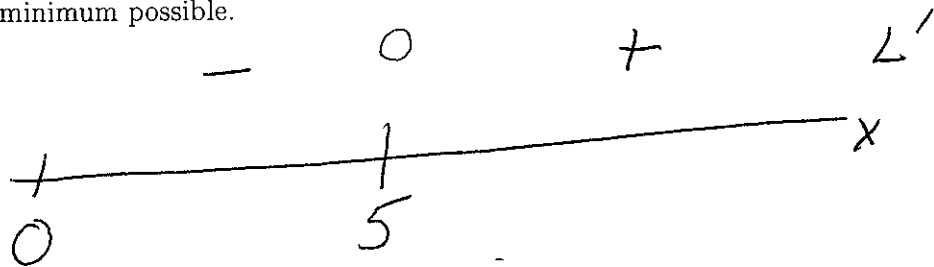
(Note: L' undefined at $x=0$.

We only consider $x > 0$)

$$y = 50(5)^{-1} = 10$$

$$\text{length} = 2(5) + 10 = \boxed{20 \text{ ft.}}$$

- (b) (5 points) Explain mathematically how you know that the length you found in part (a) is the minimum possible.



$L' < 0$ for $0 < x < 5$ and $L' > 0$

for $x > 5$, so L has a min. at $x = 5$

6. (a) (5 points) Write the formula which is used in Newton's method, giving x_{n+1} in terms of x_n . You do not need to show where the formula comes from.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- (b) (10 points) Use Newton's method to estimate $\sqrt{5}$. Carry out two steps of Newton's method. You do not need to simplify the result of the second step.

$$\text{Let } f(x) = x^2 - 5$$

$$x_0 = 2 \quad (\text{which is close to } \sqrt{5})$$

$$f'(x) = 2x$$

$$x_1 = 2 - \frac{4-5}{2 \cdot 2} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$x_2 = \frac{9}{4} - \frac{\left(\frac{9}{4}\right)^2 - 5}{2 \cdot \frac{9}{4}}$$

7. (10 points) State the Mean Value Theorem.

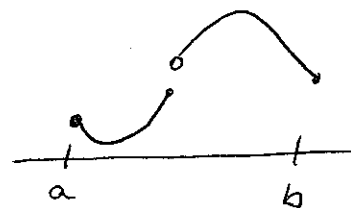
If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) ,

then there is a number c between a and b with $f'(c) = \frac{f(b) - f(a)}{b - a}$

8. (4 points each part) Answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If $f(x)$ has an absolute maximum and an absolute minimum on a closed interval $[a, b]$, then $f(x)$ must be continuous on $[a, b]$.

F. Here is a counter-example.



(b) If $f''(c) = 0$, then $f(x)$ has an inflection point at $x = c$.

F. $f(x) = x^4$ with $c = 0$ is a counterexample. Concavity must change to be inflection point.

(c) The linear approximation to $f(x)$ at $x = x_0$ is given by the equation of the tangent line to the graph of $y = f(x)$ at $x = x_0$.

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