Math 221, DL1 - Test #2 - October 22, 2007

Name:_____

Signature:_____

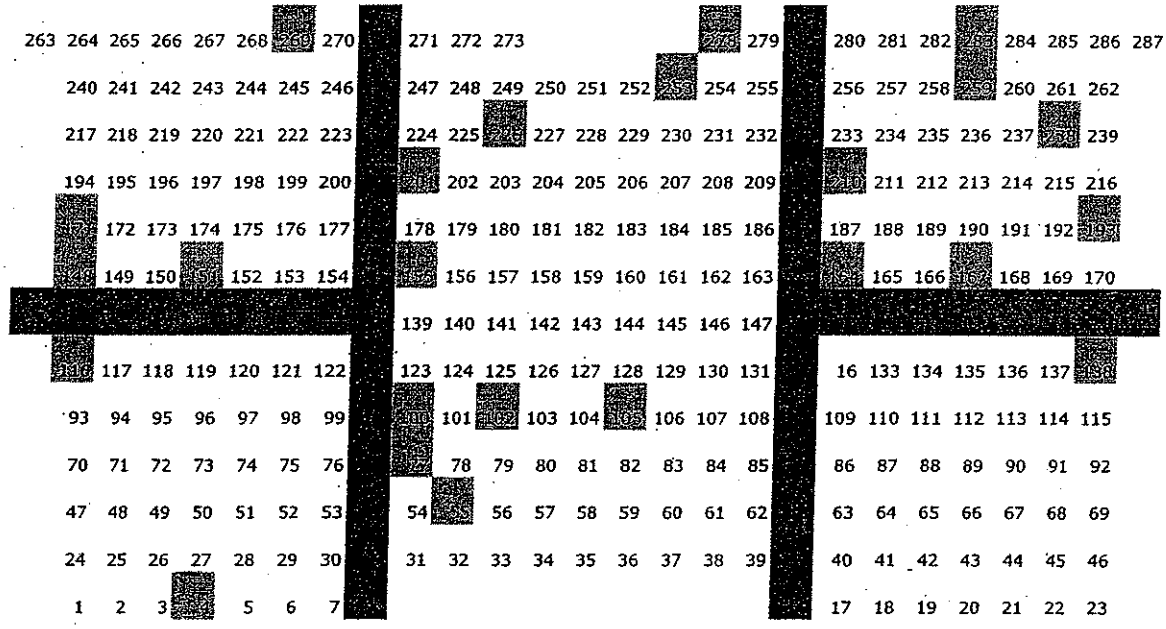
Circle your Section: DD1 (8:00, Aaron Ziegler) DD2 (9:00, John Maki)

DD3 (3:00, Aaron Ziegler) DD4 (1:00, Kevin Milans) DD5 (12:00 Kevin Milans)

DD6 (2:00, Suil O) DD7 (10:00, Suil O) DD8 (12:00, John Lenz)

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

SIT IN THE SEAT CIRCLED BELOW.



Time: 50 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.

problem number	1	2	3	4	5	6	7	8
possible points	7	10	10	16	20	15	10	12
score								

1. (7 points) Find $f'(x)$ for

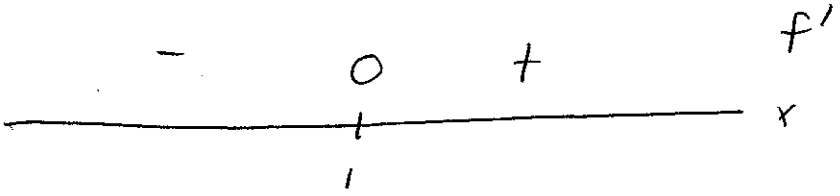
$$f(x) = \tan^{-1}(3x).$$

$$f'(x) = \frac{1}{1+(3x)^2} \cdot 3$$

2. (10 points) On which intervals is the graph of $f(x) = x^2 - 2x + 21$ increasing and on which intervals is it decreasing?

$f'(x) = 2x - 2$

$f'(x) = 0$ for $x = 1$



increasing on $(1, \infty)$

decreasing on $(-\infty, 1)$

3. (10 points) Use implicit differentiation to find dy/dx for

$$x^2 + y^2 = 25xy.$$

$$2x + 2y\frac{dy}{dx} = 25y + 25x\frac{dy}{dx}$$

$$2x - 25y = (25x - 2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - 25y}{25x - 2y}$$

4. (8 points each part) Evaluate each limit.

(a)

$$\lim_{x\to 1} \frac{x^{10}-1}{1-x^2}$$

type $\frac{0}{0}$

$$= \lim_{x\to 1} \frac{10x^9}{-2x} = \frac{10}{-2} = -5$$

(b)

$$\lim_{x\to 0^+} x \ln x$$

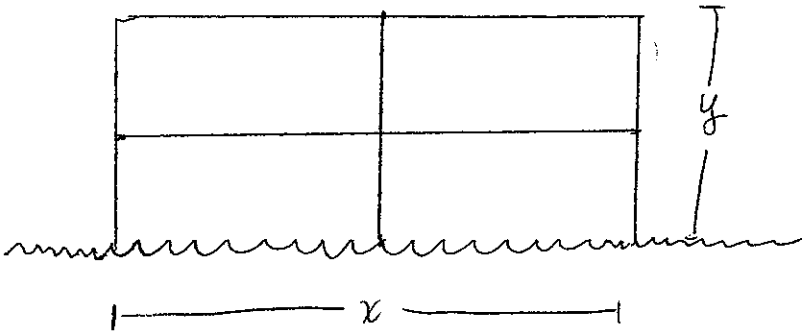
type $0 \cdot (-\infty)$

$$= \lim_{x\to 0^+} \frac{\ln x}{x^{-1}} \qquad \text{type } \frac{-\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-x^{-2}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{-x}{1} = 0$$

5. (a) (15 points) A farmer has 600 ft. of fencing with which to build a rectangular corral. A river will form one side of the corral and some of the fencing will be used to construct internal divider fences as shown. The farmer wants to make the total area of the corral as large as possible. What is the maximum total area of such a corral?



Let x and y be as shown, in feet.

Maximize area $= A = xy$

$3y + 2x = 600$, so $y = 200 - \frac{2}{3}x$

$A = x(200 - \frac{2}{3}x) = 200x - \frac{2}{3}x^2$

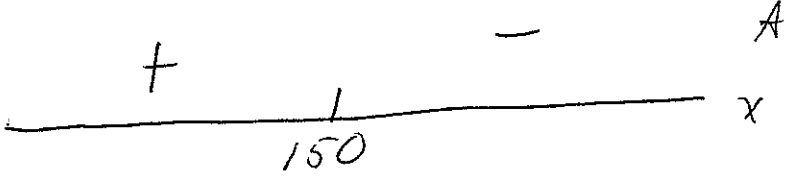
$A' = 200 - \frac{4}{3}x$

$A' = 0$ when $200 = \frac{4}{3}x$; $x = \frac{600}{4} = 150$

$y = 200 - \frac{2}{3}x = 200 - 100 = 100$

area $= 150 \cdot 100 = 15{,}000$ sq. ft.

(b) (5 points) Explain mathematically how you know that the area you found in part (a) is the maximum possible.



when $x < 150$, $A' > 0$ and when $x > 150$, $A' < 0$, so A has a maximum at $x = 150$.

6. (a) (5 points) Give the formula for the linear approximation of $f(x)$ at $x = x_0$. Your answer should be a function of x. You do not need to explain where your answer comes from.

$L(x) = f(x_0) + f'(x_0)(x - x_0)$

$\|$

y

(b) (10 points) Use linear approximation to estimate $\sqrt{17}$. Be sure to indicate what you are using for $f(x)$ and for x_0.

$f(x) = \sqrt{x}$ $\quad x_0 = 16$ $\quad x = 17$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(x_0) = 4$

$f'(x_0) = \frac{1}{8}$

$L(17) = 4 + \frac{1}{8}(17 - 16)$

$= 4 + \frac{1}{8} = \frac{33}{8}$

$\sqrt{17}$ is approximately $\frac{33}{8}$.

7. (10 points) State the Extreme Value Theorem.

If f is continuous on $[a,b]$ then f has an absolute max and an absolute min on $[a,b]$

8. (4 points each part) Answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

   (a) If $f(x)$ is continuous and differentiable for all x, and if $f(1) = 2$ and $f(2) = 5$, then there must be a number c with $f'(c) = 3$.

T from the Mean Value Theorem

   (b) If $f'(-1) = 0$ and $f''(-1) = 0$, then $f(x)$ has neither a local maximum nor a local minimum at $x = -1$.

F It could have a local max or local min or neither. More information is needed.

   (c) When using Newton's method, we must start with an initial guess x_0 for which $f(x_0) = 0$.

F. If we already have x_0 with $f(x_0)=0$ then we don't need Newton's method and if we use it, it will give $x_n = x_0$ at each step.