Math 230, BL1 - Final Exam - May 6, 2006

DO NOT OPEN EXAM UNTIL TOLD TO DO SO

Name:____

Signature:____

Circle your Recitation Section:

8:00 (BD1, Hailong Hu)

9:00 (BD2, Hailong Hu)

11:00 (BD3, Caleb Eckhardt)

12:00 (BD4, Caleb Eckhardt)

12:00 (BD7, Carolyn Wendler)

1:00 (BD5, Carolyn Wendler)

3:00 (BD6, Bin Wang)

Time: 3 hours. You may not use any books or notes or calculator. There are 200 points possible. To get full credit, you must show your work. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.

problem	1	2	3	4	5	6	7	8
possible	11	11	11	11	11	10	11	11
score								

problem	9	10	11	12	13	14	15	16
possible	11	12	14	12	20	15	15	14
score								

1. (11 points) Evaluate

$$\int x \ln x \, dx.$$

2. (11 points) Evaluate

$$\int \tan 5x \, dx.$$

3. (11 points) Evaluate

$$\int \frac{2x^2 + 14x + 49}{x^3 - 7x^2}\, dx.$$

4. (11 points) Evaluate

$$\int \frac{1}{\sqrt{x^2+25}}\,dx.$$

5. (11 points) Determine whether the following improper integral converges or diverges. If it converges, find its value.

$$\int_0^\infty xe^{-x^2}\,dx$$

6. (a) (5 points) Find the 3rd partial sum S_3 for the series

$$\sum_{n=1}^{\infty} \frac{1}{2^n}.$$

(b) (5 points) If we know that for a given series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums satisfies $\lim_{n\to\infty} S_n = 4$, can we conclude that $\sum_{n=1}^{\infty} a_n$ converges? Explain your answer.

7. (11 points) Determine whether the infinite series below converges or diverges. If using a test, state which test you are using.

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots + \frac{2^n}{3^n} + \cdots$$

8. (11 points) Determine whether the infinite series below converges or diverges. Suggestion: integral test.

$$\sum_{n=2}^{\infty} \frac{1}{n\, \ln n}$$

9. (11 points) Determine whether the infinite series below converges or diverges. If using a test, state which test you are using.

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$$

10. (12 points) Find the 3rd degree (that is, n=3) Taylor polynomial with remainder for $f(x) = \sin x$, centered at $x = \pi/6$. Be sure to indicate which is the Taylor polynomial and whichis the remainder. In the remainder, be sure to state what the restrictions are on z. (Note: $\cos(\pi/6) = \sqrt{3}/2$ and $\sin(\pi/6) = 1/2$)

11. (14 points) Find a general solution to the differential equation

$$\frac{dy}{dx} = 2y.$$

Then find a particular solution that also satisfies the initial condition $y(1) = 3$.

12. (12 points) Suppose that f is a function with power series

$$f(x) = 4x + 3x^2 + \frac{4}{3}x^3 + \cdots + \frac{2n+2}{n!}x^n + \cdots$$

Use this power series to evaluate

$$\lim_{x \to 0} \frac{f(x)}{x}.$$

13. (a) (5 points) Find the rectangular coordinates (x, y) for the point with polar coordinates $(r, \theta) = (2, -\pi/4)$.

(b) (5 points) Find polar coordinates (r, θ) for the point with rectangular coordinates $(x, y) = (0, -4)$.

(c) (10 points) On the xy-coordinate plane, sketch the graph of the polar equation

$$r = 1 - \sin\theta.$$

14. (15 points) Find the area bounded by one loop of the curve

$$r = 2\cos 2\theta.$$

15. (15 points) Find the (x, y) coordinates of the points on the parametric curve $x = 3\cos t$, $y = 4\sin t$ where the tangent line is horizontal.

16. (14 points) Consider the region that lies between the parametric curve

$$x = \cos t, \quad y = \sin^2 t, \quad 0 \le t \le \pi.$$

Find the volume obtained by revolving this region around the x-axis.