Name of test: The Root Test

What type of series does it apply to?:

Any series $\Sigma\, a_n$ – no condition on a_n.

Hypotheses of the test (the "if" part):

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$, then the series converges absolutely
If $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $= \infty$, then the series diverges
If the limit =1 or does not exist, then the test gives no information

Conclusion of the test (the "then" part):

See above under "Hypotheses".

When would this test be a good one to try?

When a_n is some expression raised to the n-th power.

Example for which the test shows convergence (if applicable):

$\sum_{n-1}^{\infty} \frac{(\ln n)^n}{n^n}$ For this example, $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 0$, so the test tells us that the series converges.

Example for which the test shows divergence (if applicable):

$\sum_{n=1}^{\infty} \frac{2^{n^2}}{n^n}$ In this example, $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{2^n}{n} = \infty$, so the test tells us that the series diverges.

Example for which the test gives no conclusion:

$\sum_{n=1}^{\infty} \left(\frac{n-2}{n}\right)^n$ For this example, $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, so the test gives no conclusion. One can instead see that this series diverges by using the n-th term test.

Is there an error estimate associated with this test? What is it? No error estimate.