13.8 Problems

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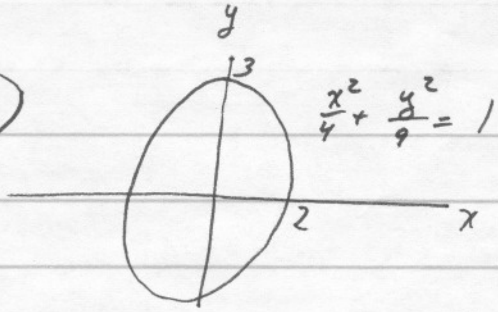


$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$z = 3y + x$$

The integrand is $\sqrt{1 + z_x^2 + z_y^2} = \sqrt{1 + 1^2 + 3^2} = \sqrt{11}$

R is the ellipse shown above $\quad y = \pm 3\sqrt{1 - \frac{x^2}{4}}$

surface area $= \iint_R \sqrt{11}\, dA = \int_{x=-2}^{x=2} \int_{y=-3\sqrt{1-x^2/4}}^{y=3\sqrt{1-x^2/4}} \sqrt{11}\, dy\, dx$

$$= 6\sqrt{11} \int_{x=-2}^{x=2} \sqrt{1 - \frac{x^2}{4}}\, dx$$

Let $x = 2\cos\theta$ (trig subs.). $\quad dx = -2\sin\theta\, d\theta$

$x = 2 \Rightarrow \theta = 0 \qquad x = -2 \Rightarrow \theta = \pi$

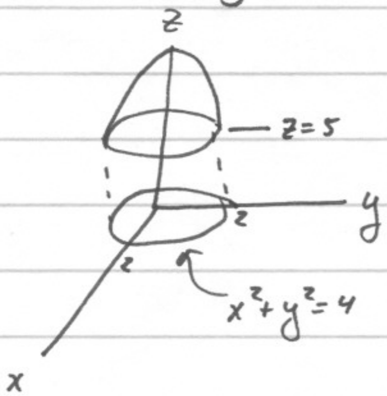
$$= 6\sqrt{11} \int_{\theta=\pi}^{\theta=0} \sin\theta\,(-2\sin\theta)\, d\theta = -12\sqrt{11} \int_{\theta=\pi}^{\theta=0} \tfrac{1}{2}(1 - \cos 2\theta)\, d\theta$$

$$= -6\sqrt{11}\left(\theta - \tfrac{1}{2}\sin 2\theta\right)\Big|_{\pi}^{0} = 6\sqrt{11}(\pi - 0) = \boxed{6\sqrt{11}\,\pi}$$

~~13.8 Prob~~

13.8/3) Intersection of plane and paraboloid:

$5 = 9 - x^2 - y^2 \qquad x^2 + y^2 = 4$. circle.



$z = 9 - x^2 - y^2 = 9 - r^2$ in cylindrical coords.

$z_r = -2r \quad ; \quad z_\theta = 0$

integrand $= \sqrt{r^2 + (r z_r)^2 + (z_\theta)^2} = \sqrt{r^2 + 4r^4} = r\sqrt{1+4r^2}$

$$\text{Surface Area} = \int_{\theta=0}^{2\pi} \int_{r=0}^{r=2} r\sqrt{1+4r^2}\, dr\, d\theta$$

$$= \frac{1}{8} \cdot \frac{2}{3} \int_{\theta=0}^{2\pi} (1+4r^2)^{3/2} \Big|_{r=0}^{r=2} d\theta$$

$$= \frac{1}{12} \int_{\theta=0}^{2\pi} (17)^{3/2} - 1 \, d\theta = \frac{1}{12}\left((17)^{3/2} - 1\right) \cdot 2\pi$$

$$= \frac{\pi}{6}\left((17)^{3/2} - 1\right)$$

(13.8/5) $z = x + y^2$. Integrand $= \sqrt{1 + 1^2 + (2y)^2}$
$= \sqrt{2 + 4y^2}$

$$\text{Surface Area} = \int_{x=0}^{x=1} \int_{y=0}^{y=2} \sqrt{2+4y^2}\, dy\, dx$$

trig subs: let $y = \frac{1}{\sqrt{2}} \tan\theta$ or use formula (44) from the book's integral table.

$$\int \sqrt{2+4y^2} = 2\int \sqrt{\tfrac{1}{2} + y^2}\, dy = \qquad \text{(formula (44))}$$

$$\frac{y}{2}\sqrt{\tfrac{1}{2} + y^2} + \frac{1}{4} \ln\left| y + \sqrt{\tfrac{1}{2} + y^2} \right|$$

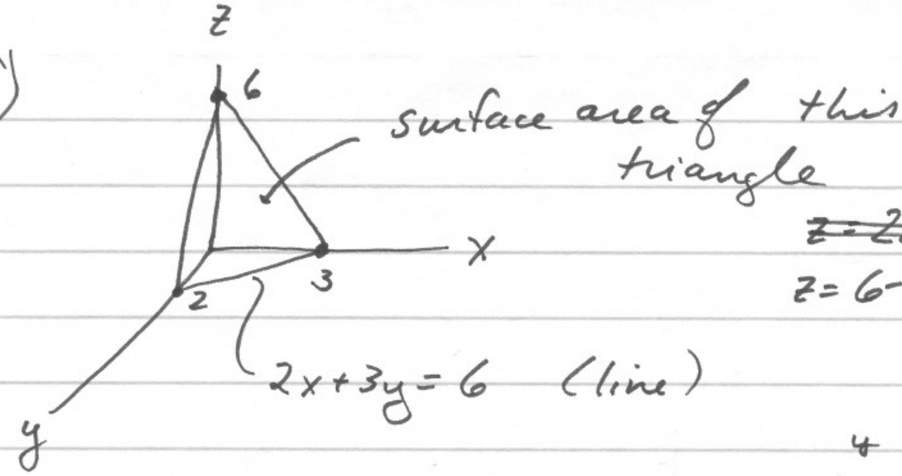
$$\text{so S.A.} = 2\int_{x=0}^{x=1} \left[\sqrt{\tfrac{1}{2} + 4} + \frac{1}{4}\ln\left| 2 + \sqrt{\tfrac{1}{2}+4} \right| - \frac{1}{4} \ln \sqrt{\tfrac{1}{2}} \right] dx$$

$$= 2\left[\frac{3}{\sqrt{2}} + \frac{1}{4}\ln\left(2 + \frac{3}{\sqrt{2}}\right) - \frac{1}{4}\ln\left(\frac{1}{\sqrt{2}}\right) \right]$$

$$= 2\left[\frac{3}{\sqrt{2}} + \frac{1}{4}\ln\left(2\sqrt{2} + 3\right) \right]$$

$$= 3\sqrt{2} + \frac{1}{2}\ln(2\sqrt{2} + 3)$$

13.8/7

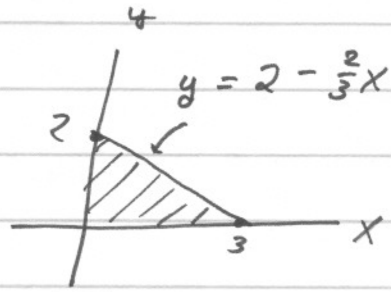


~~$z = 2x + 3y + 1$~~

$z = 6 - 2x - 3y$

region over which we integrate:

$0 \le x \le 3$; $0 \le y \le 2 - \frac{2}{3}x$



Integrand $= \sqrt{1 + (-2)^2 + (-3)^2} = \sqrt{14}$

Surface area $= \displaystyle\int_{x=0}^{x=3} \int_{y=0}^{y=2-\frac{2}{3}x} \sqrt{14}\, dy\, dx$

$= \sqrt{14} \displaystyle\int_{x=0}^{x=3} 2 - \frac{2}{3}x \, dx = \sqrt{14}\left[2x - \frac{1}{3}x^2 \Big|_0^3\right]$

$= \sqrt{14}\left[6 - 3\right] = \boxed{3\sqrt{14}}$

13.8/13 $x = a\cos\theta$; $y = a\sin\theta$; $z = z$

$\bar{r}\,\cancel{N}(\theta, z) = \langle a\cos\theta,\ a\sin\theta,\ z\rangle$

$$\frac{\partial \vec{r}}{\partial \theta} = \langle -a\sin\theta,\ a\cos\theta,\ 0\rangle$$

$$\frac{\partial \vec{r}}{\partial z} = \langle 0,\ 0,\ 1\rangle$$

$\cancel{N(u,v)}$ $\vec{N}(\theta, r) = \frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial z}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a\sin\theta & a\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i}\,(a\cos\theta) - \vec{j}\,(-a\sin\theta) + \vec{k}\,(0)$$

$$= \langle a\cos\theta,\ a\sin\theta,\ 0\rangle$$

$$|\vec{N}(\theta, r)| = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta} = a$$

$$\text{Surface Area} = \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z=h} a\, dz\, d\theta = \boxed{2\pi a h}$$