

1	2	3	4	5	6

Solutions

Name: _____

Circle your Recitation Section: 8:00 10:00 12:00 2:00 3:00

Math 242, Section BL1 - Test #3 - December 7, 2005

Time: 55 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (16 points) Calculate the Riemann sum for

$$\iint_R f(x, y) dA,$$

using the given partition and selection of points (x_i^*, y_i^*) for the rectangle R .

$$f(x, y) = x + y;$$

$$R = [0, 2] \times [0, 2];$$

the partition P consists of four unit squares;

each (x_i^*, y_i^*) is the center point of the i th rectangle R_i .

See version 1

2. (15 points) Evaluate the iterated integral

$$\int_0^1 \int_0^{y^3} xy \, dx \, dy.$$

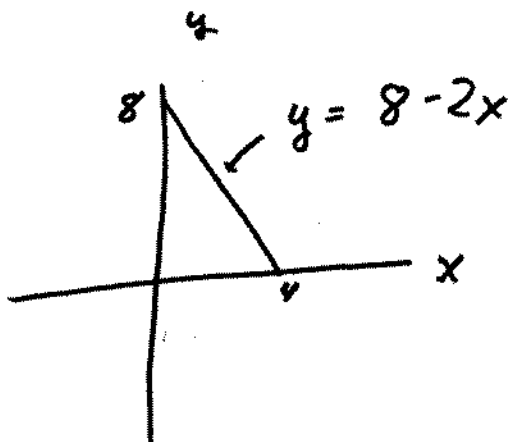
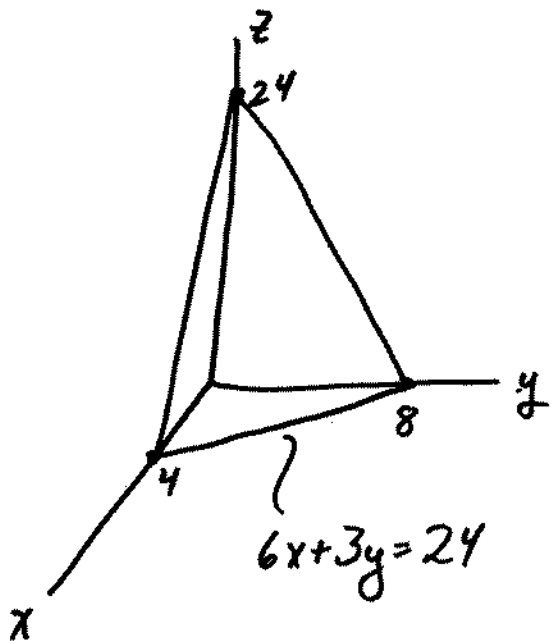
$$= \int_{y=0}^{y=1} \int_{x=0}^{x=y^3} \frac{1}{2} x^2 y \Big|_{x=0}^{x=y^3} dy$$

$$= \int_{y=0}^{y=1} \frac{1}{2} y^7 dy$$

$$= \frac{1}{16} y^8 \Big|_0^1$$

$$= \frac{1}{16}$$

3. (16 points) Set up an iterated integral which gives the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $6x + 3y + z = 24$. Do not evaluate the integral.



$$Vd = \int_{x=0}^{x=4} \int_{y=0}^{y=8-2x} \int_{z=0}^{z=24-6x-3y} 1 \, dz \, dy \, dx$$

or

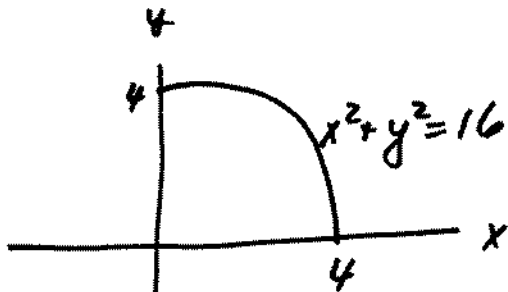
$$\int_{x=0}^{x=4} \int_{y=0}^{y=8-2x} 24-6x-3y \, dy \, dx$$

or

$$\int_{y=0}^{y=8} \int_{x=0}^{x=4-\frac{1}{2}y} 24-6x-3y \, dx \, dy$$

4. (20 points) Convert the following integral to polar coordinates and evaluate.

$$\int_{x=0}^{x=4} \int_{y=0}^{y=\sqrt{16-x^2}} y \, dy \, dx$$



$$y = r \sin \theta$$

$$\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=4} r^2 \sin \theta \, dr \, d\theta \quad 10$$

$$= \int_{\theta=0}^{\theta=\pi/2} \left. \frac{1}{3} r^3 \sin \theta \right|_{r=0}^{r=4} d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/2} \frac{64}{3} \sin \theta \, d\theta$$

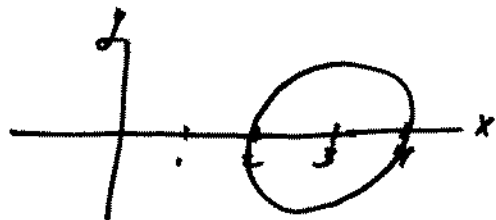
$$= \frac{-64}{3} \cos \theta \Big|_0^{\pi/2} = \frac{-64}{3} (0 - 1) = \frac{64}{3}$$

5. Let R be the circle $(x - 3)^2 + y^2 \leq 1$, with density function $\delta(x, y) = 1$. Let T be the solid torus which is obtained by revolving R around the y -axis.

(a) (5 points) State the 1st Theorem of Pappus. Be sure to indicate what your notation stands for.

For a solid generated by revolving a region R around a line, $V = A \cdot d$, where $V =$ volume of solid,
 $A =$ area of R , $d =$ distance travelled by centroid of R

(b) (5 points) What is the centroid of R ? Don't use any integrals - just use common sense and symmetry.



$(\bar{x}, \bar{y}) = (3, 0)$, the center of the circle

(c) (2 points) What is the area of the circle R ? (Don't compute any integrals - just use the formula for the area of a circle).

$$\pi r^2 = \pi$$

(d) (2 points) How far does the centroid of R travel when revolved around the y -axis?

$$2\pi \cdot 3 = 6\pi$$

(e) (3 points) Use the 1st Theorem of Pappus to find the volume of the torus T .

$$V = \pi \cdot 6\pi = 6\pi^2$$

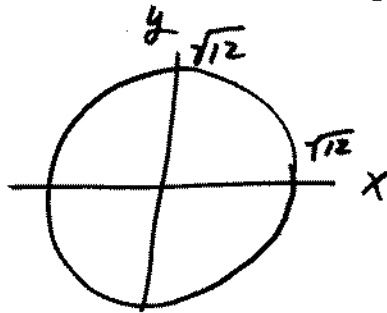
17 total

6. (16 points) Let T be the solid bounded by the elliptic paraboloids $z = 2x^2 + 3y^2$ and $z = 48 - 2x^2 - y^2$, with density function $\delta(x, y, z) = x^2 + y^2$. Set up a triple iterated integral which equals the mass of T . Do not evaluate the integral.

intersection: $2x^2 + 3y^2 = 48 - 2x^2 - y^2$

$$4x^2 + 4y^2 = 48$$

$$x^2 + y^2 = 12$$



mass = $\int_{x=-\sqrt{12}}^{x=\sqrt{12}} \int_{y=-\sqrt{12-x^2}}^{y=\sqrt{12-x^2}} \int_{z=2x^2+3y^2}^{z=48-2x^2-y^2} (x^2 + y^2) dz dy dx$