Homework #3 Solutions to some problems

242 Fall '05

11.4/4 $x(t) = 17 - 17t$; $y(t) = -13 + 13t$; $z(t) = -31 + 31t$

11.4/6 direction vector $\overrightarrow{P_2P_1} = \langle 3, -13, 3 \rangle$

Using P_1 as the point,

$\boxed{x(t) = 3 + 3t \; ; \; y(t) = 5 - 13t \; ; \; z(t) = 7 + 3t}$

Note: if you use P_2 instead of P_1, you'll get different parametric equations which are also correct.

11.4/10 direction vector $\overrightarrow{PQ} = \langle 2, -2, 15 \rangle$

Using P, parametric eqs.

$\boxed{x(t) = 2 + 2t \; ; \; y(t) = 5 - 2t \; ; \; z(t) = -7 + 15t}$

Solve for t: $t = \frac{x-2}{2}$ $t = \frac{y-5}{-2}$ $t = \frac{z+7}{15}$

$\boxed{\frac{x-2}{2} = \frac{y-5}{-2} = \frac{z+7}{15}}$ symmetric eqs.

11.4/16 The direction vector for L_1 is $\langle 4, 1, -2 \rangle$

The direction vector for L_2 is $\langle 6, -3, 8 \rangle$

These are not scalar multiples of one another, so L_1 and L_2 are $\boxed{\text{not parallel.}}$

Parametric Eqs. for L_1 are

$x = 11 + 4t$; $y = 6 + t$; $z = -5 - 2t$

And for L_2

$x = 13 + 6s$; $y = 2 - 3s$; $z = 5 + 8s$

If L_1, L_2 intersect, then

$$\begin{cases} 11 + 4t = 13 + 6s & (1) \\ 6 + t = 2 - 3s & (2) \\ -5 - 2t = 5 + 8s & (3) \end{cases}$$

Solve (2) for t and plug into (3) to get

$$-5 - 2(-4 - 3s) = 5 + 8s$$

$$3 - 3s = 5 + 8s$$

$$-2 = 11s$$

$$s = -2/11$$

Then from (2),

$$t = -4 - 3(-2/11)$$

$$= -4 + \frac{6}{11} = \frac{-38}{11}$$

Put this s, t into (1):

$$11 + 4\left(\frac{-38}{11}\right) = 13 + 6\left(-2/11\right)$$

This is <u>false</u> (work it out).

There is <u>no</u> point of intersection; $\boxed{\text{skew}}$

11.4/20 The direction vector for L_1 is $\langle 12, 20, -28 \rangle$

And for L_2 $\langle 9, 15, -21 \rangle$

These are parallel because $\langle 12, 20, -28 \rangle = \frac{4}{3} \langle 9, 15, -21 \rangle$

L_1 and L_2 are $\boxed{\text{parallel}}$.

11.4/22 $\vec{n} \cdot (\vec{r} - \langle 3, -4, 5 \rangle) = 0$ Let $\vec{r} = \langle x, y, z \rangle$

$$\boxed{-2(x-3) + 7(y+4) + 3(z-5) = 0.}$$

11.4/30 The normal vector to $x + y - 2z = 0$

is $\vec{n} = \langle 1, 1, -2 \rangle$.

Answer: $\boxed{(x-5) + (y-1) - 2(z-4) = 0}$

11.4/37 For L, the direction vector is $\vec{v} = \langle 2, -5, 3 \rangle$

For $\mathcal{P}$, the normal vector is $\vec{n} = \langle 3, 2, -4 \rangle$

Since $\vec{n} \cdot \vec{v} \neq 0$, $\vec{v}$ and $\vec{n}$ are not perpendicular, so L is not parallel to $\mathcal{P}$.

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11.4/37 cont'd

To find the intersection, plug the equations for L into the equation for $\mathcal{P}$:

$$3(3+2t)+2(6-5t)-4(2+3t)=1.$$

$$9+6t+12-10t-8-12t=1$$

$$13-16t=1 \qquad -16t=-12 \qquad t=\tfrac{12}{16}=\tfrac{3}{4}$$

Using the equations for L,

$$x=3+2(\tfrac{3}{4})=3+\tfrac{3}{2}=\tfrac{9}{2}$$

$$y=6-5(\tfrac{3}{4})=6-\tfrac{15}{4}=\tfrac{9}{4}$$

$$z=2+3(\tfrac{3}{4})=2+\tfrac{9}{4}=\tfrac{17}{4}$$

$\boxed{\left(\frac{9}{2},\frac{9}{4},\frac{17}{4}\right)}$ is the point of intersection

11.4/40 The normal vectors are

$$\vec{n}=\langle 2,-1,1\rangle, \quad \vec{m}=\langle 1,1,-1\rangle$$

$\vec{n}\cdot\vec{m}=2-1-1=0$, so $\vec{n}\perp\vec{m}$.

The angle is $90°$ or $\boxed{\pi/2 \text{ radians}}$.

11.4/44 To find a point on the intersection of the two planes, let $z=0$ (this is arbitrary - any z works)

Then $2x-y=5$ and $x+y=1$.

Solving for x and y, we get $x=2$, $y=-1$

So $P_0 = (2,-1,0)$ is on the line of intersection.

The direction vector is parallel to both planes, so it is perpendicular to both $\vec{n}$ and $\vec{m}$ (see #40 for notation). We get it by taking $\vec{n} \times \vec{m}$:

$$\vec{n} \times \vec{m} = \langle 2,-1,1\rangle \times \langle 1,1,-1\rangle$$

$$= \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \vec{k}$$

$$= 0\vec{i} + 3\vec{j} + 3\vec{k} = \langle 0,3,3\rangle$$

Parametric eqs of line: $\boxed{x=2,\quad y=-1+3t,\quad z=0+3t}$

Symmetric eqs: $\boxed{x=2,\quad \dfrac{y+1}{3} = \dfrac{z}{3}}$

11.4/51 Plan: ① Find the direction vector $\vec{v}$ of the intersection of the given planes. ② Then $\overrightarrow{PQ}$ and $\vec{v}$ are both perpen. to the normal vector $\vec{n}$ of the plane we seek, so $\vec{n} = \overrightarrow{PQ} \times \vec{v}$. ③ Find the equation by using P and $\vec{n}$.

① The normal vectors of the given planes are $\vec{m}_1 = \langle 1,1,1\rangle$, $\vec{m}_2 = \langle 3,-1,0\rangle$. The direction vector $\vec{v}$ of their intersection is

$$\vec{v} = \vec{m}_1 \times \vec{m}_2 = \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} \vec{k}$$

$$= \vec{i} + 3\vec{j} - 4\vec{k} = \langle 1, 3, -4\rangle$$

② $\vec{n} = \overrightarrow{PQ} \times \vec{v} = \langle 1,1,1\rangle \times \langle 1,3,-4\rangle$

$$= \begin{vmatrix} 1 & 1 \\ 3 & -4 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 1 & -4 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} \vec{k}$$

$$= -7\vec{i} + 5\vec{j} + 2\vec{k} = \langle -7, 5, 2\rangle$$

③ Equation of a plane containing P and perpendicular to $\langle -7, 5, 2\rangle = \vec{n}$ is

$$\boxed{-7(x-1) + 5y + 2(z+1) = 0}$$

11.4/52 Parametric equations for the lines:

$$t = x-1 \qquad x = t+1$$
$$t = \tfrac{1}{2}(y+1) \qquad y = 2t-1$$
$$t = z-2 \qquad z = t+2$$

and

$$s = x-2 \qquad x = s+2$$
$$s = \tfrac{1}{3}(y-2) \qquad y = 3s+2$$
$$s = \tfrac{1}{2}(z-4) \qquad z = 2s+4$$

To find the point of intersection:

$$\begin{cases} t+1 = s+2 & (1) \\ 2t-1 = 3s+2 & (2) \\ t+2 = 2s+4 & (3). \end{cases}$$

Using (1) and (3), we get $s=-1$, $t=0$. This works in (2) also.

Putting $t=0$ into the parametric equations for the first line gives $(x,y,z) = (1,-1,2)$ as the point of intersection

The direction vectors of the two lines are $\vec{v}_1 = \langle 1,2,1 \rangle$, $\vec{v}_2 = \langle 1,3,2 \rangle$.

Then $\vec{v}_1 \times \vec{v}_2$ is the normal vector of the plane containing the two lines.

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \vec{k}$$

$$= \vec{i} - \vec{j} + \vec{k} = \langle 1,-1,1 \rangle$$

The equation of the plane is

$$1\cdot(x-1) - 1(y+1) + 1(z-2) = 0 \quad \text{or} \quad \boxed{x-y+z = 4}$$