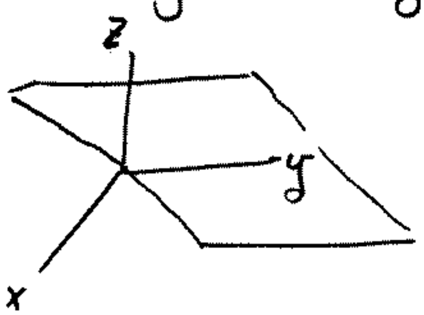
242 Homework #6 Solutions (plus a few extras)

(12.2/2) $f(x,y) = \sqrt{x^2 + 2y^2}$ All (x,y), since $x^2 + 2y^2$ is never negative.

(12.2/4) $f(x,y) = \dfrac{1}{x-y}$ All (x,y) with $x \neq y$

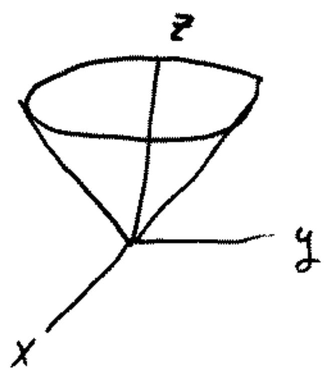
(12.2/6) all (x,y) with $x \geq 0$

(12.2/22) $f(x,y) = x$. This is the plane given by $z = x$ ($y =$ anything)



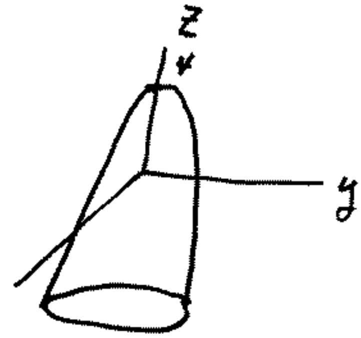
(12.2/24) $z = \sqrt{x^2 + y^2}$. $z^2 = x^2 + y^2$ $(z \geq 0)$

This is a cone

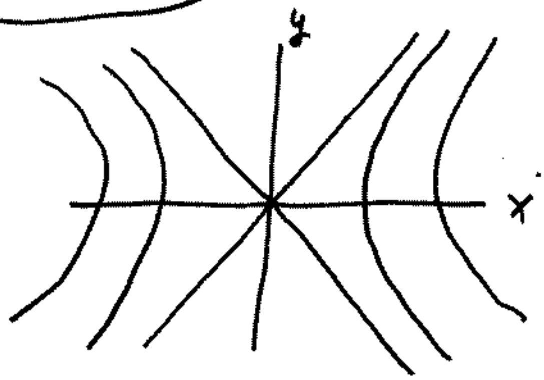


See p. 835. Since z is defined as $\sqrt{x^2 + y^2}$, $z \geq 0$, so we have the top half of the picture on p. 835

12.2/26 $z = 4 - x^2 - y^2$. This is a paraboloid opening downwards.

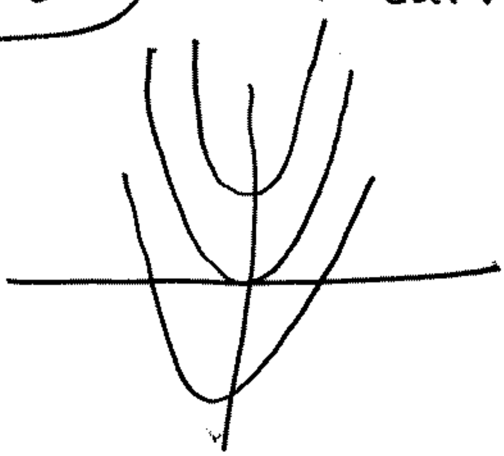


12.2/32 Level curves $x^2 - y^2 = k$. hyperbolas



12.2/34 Level curves $y - x^2 = k$ $y = k + x^2$

parabolas



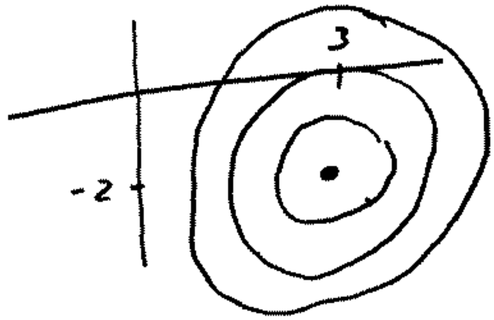
All have the same shape, different y-intercept.

12.2/38 $\cancel{z}^{k} = x^2 + y^2 - 6x + 4y + 7$

$k - 7 + 9 + 4 = (x^2 - 6x + 9) + (y^2 + 4y + 4)$

$k + 6 = (x-3)^2 + (y+2)^2$

Circles with center $(3, -2)$ (for $k \geq -6$)



12.2/44 $k = z^2 - x^2 - y^2$. These level surfaces are hyperboloids of two sheets for $k > 0$ and hyperboloids of one sheet for $k < 0$. For $k = 0$, the level surface is a double cone like shown on p. 835.

12.2/53 - 12.2.41

12.2/54 - 12.2.39

12.2/55 - 12.2.42

12.2/56 - 12.2.40

12.2/57 - 12.2.44

12.2/58 - 12.2.43

12.3/17 $\lim\limits_{h\to 0} \dfrac{f(x+h,y)-f(x,y)}{h} = \lim\limits_{h\to 0} \dfrac{(x+h)y - xy}{h}$

$$= \lim_{h\to 0} \frac{xy+hy-xy}{h} = \lim_{h\to 0} \frac{hy}{h} = \lim_{h\to 0} y = y$$

$$\lim_{k\to 0} \frac{f(x,y+k)-f(x,y)}{k} = \lim_{k\to 0} \frac{x(y+k)-xy}{k}$$

$$= \lim_{k\to 0} \frac{xy+xk-xy}{k} = \lim_{k\to 0} \frac{xk}{k} = \lim_{k\to 0} x = x$$

12.3/18 $\lim\limits_{h\to 0} \dfrac{f(x+h,y)-f(x,y)}{h}$

$$= \lim_{h\to 0} \frac{(x+h)^2+y^2-(x^2+y^2)}{h} = \lim_{\cancel{k}h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} 2x+h = 2x$$

$$\lim_{k\to 0} \frac{f(x,y+k)-f(x,y)}{k}$$

$$= \lim_{k\to 0} \frac{x^2+(y+k)^2-(x^2+y^2)}{k}$$

$$= \lim_{k\to 0} \frac{2yk+k^2}{k} = \lim_{k\to 0} 2y+k = 2y$$

12.3/38 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$

$$= \lim_{r\to 0} \frac{r^3\cos^3\theta - r^3\sin^3\theta}{r^2}$$

$$= \lim_{r\to 0} r(\cos^3\theta - \sin^3\theta) = 0$$

since $\cos^3\theta - \sin^3\theta$ is bounded between -2 and 2.

12.3/41 $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$

$$= \lim_{\rho\to 0} \frac{\rho\sin\phi\cos\theta\,\rho\sin\phi\sin\theta\,\rho\cos\phi}{\rho^2}$$

$$= \lim_{\rho\to 0} \rho(\sin^2\phi\cos\theta\sin\theta\cos\phi) = 0$$

since sin and cos are bounded between -1 and 1.

12.3/43 Let $y = mx$. Then

$$\frac{x^2-y^2}{x^2+y^2} = \frac{(1-m^2)x^2}{(1+m^2)x^2} = \frac{1-m^2}{1+m^2}.$$

So as we approach $(0,0)$ along the path $y=mx$, the limit is $\frac{1-m^2}{1+m^2}$. This is different for different m, so the given limit does not exist.

12.4/2 $\frac{\partial f}{\partial x} = \sin y \qquad \frac{\partial f}{\partial y} = x\cos y$

12.4/4 $\frac{\partial f}{\partial x} = e^{z} y e^{xy} \qquad \frac{\partial f}{\partial y} = e^{z} x e^{xy}$

12.4/6 $\frac{\partial f}{\partial x} = \frac{y(x^2+y^2) - xy\,(2x)}{(x^2+y^2)^2} = \frac{y^3 - x^2 y}{(x^2+y^2)^2}$

$$\frac{\partial f}{\partial y} = \frac{x(x^2+y^2) - xy\,(2y)}{(x^2+y^2)^2} = \frac{x^3 - xy^2}{(x^2+y^2)^2}.$$

12.4/8 $\frac{\partial f}{\partial x} = 14(x-y)^{13} \qquad \frac{\partial f}{\partial y} = 14(x-y)^{13}(-1)$

12.4/12 $\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 3y^2 \qquad \frac{\partial f}{\partial z} = 4z^3$

12.4/14 $\frac{\partial f}{\partial x} = 4x^3 \qquad \frac{\partial f}{\partial y} = -16z \qquad \frac{\partial f}{\partial z} = -16y$

12.4/22 $z = 2x^3 + 5x^2y - 6y^2 + xy^4$

$z_x = 6x^2 + 10xy + y^4$; $z_{xy} = 10x + 4y^3$

$z_y = \cancel{10xy}\ 5x^2 - 12y + 4xy^3$; $z_{yx} = 10x + 4y^3$.

so $z_{xy} = z_{yx}$

12.4/31 $z = x^2 + y^2$. $z_x = 2x$, $z_y = 2y$

$P = (3, 4, 25)$ $z_x = 6$, $z_y = 8$ at P

Using formula (11) on p. 874, the equation of the tangent plane is

$$z - 25 = 6(x-3) + 8(y-4)$$

12.4/65 $z = x^2 + 2xy + 2y^2 - 6x + 8y$

$z_x = 2x + 2y - 6$ $z_y = 2x + 4y + 8$

The tangent plane is horizontal if $z_x = 0$, $z_y = 0$

$$\begin{cases} 2x + 2y - 6 = 0 \\ 2x + 4y + 8 = 0 \end{cases}$$

Subtract the first eq. from the second eq.

$2y + 14 = 0$ $y = -7$

substituting in, $x = 10$. Answer: $(10, -7, -58)$

Then $z = -58$