242 Homework #7 Solutions

(12.5/4) $f(x,y) = x^2+y^2+2x$

$f_x = 2x+2, \quad f_y = 2y$

Setting $f_x=0$ and $f_y=0$ we get

$x=-1, \quad y=0. \quad f(-1,0) = 1+0-2 = -1$

Answer: $(x,y,z) = (-1,0,-1)$

~~(12.5/19)~~ $\cancel{f(x,y) = 2x^2+8xy+y^4}$

$\cancel{f_x = 4x+8y = 4(x+2y)}$

$\cancel{f_y = 8x+4y = 4(2x+y).}$

~~Setting~~ $\cancel{f_x=0, \quad f_y=0,}$ ~~we get~~

$\cancel{\left.\begin{matrix} x+2y=0 \\ 2x+y=0. \end{matrix}\right\}}$ ~~$x=0, y=0$ is the only solution.~~ $\cancel{f(0,0)=0.}$

Answer: $(x,y,z) = (0,0,0)$

(12.5/12) $f(x,y) = 2xy\,e^{-\frac{4x^2+y^2}{8}}$

$$f_x = 2y\,e^{-\frac{1}{8}(4x^2+y^2)} + 2xy\,e^{-\frac{1}{8}(4x^2+y^2)}(-x)$$

$$= 2y\,e^{-\frac{1}{8}(4x^2+y^2)}(1-x^2)$$

$$f_y = 2x\,e^{-\frac{4x^2+y^2}{8}} + 2xy\,e^{-\frac{4x^2+y^2}{8}}\left(-\tfrac{1}{4}y\right)$$

$$= \tfrac{1}{2}x\,e^{-\frac{4x^2+y^2}{8}}\left[4-y^2\right]$$

cont'd →

12.5/12 cont'd. Setting f_x and $f_y = 0$

and noting $e^u \neq 0$, we get

$$\begin{cases} y(1-x^2) = 0 & (1) \\ x(4-y^2) = 0 & (2) \end{cases}$$

If $x=0$, then (2) is true and from (1) we get $y(\cancel{1}-0) = 0$ so $y = 0$.

If $x \neq 0$, then from (2) we get $y = 2$ or -2. Putting $y = \pm 2$ into (1), we get $1 - x^2 = 0$ so $x = \pm 1$.

x	y	$z = f(x,y)$
0	0	0
1	2	$4e^{-1}$
-1	2	$-4e^{-1}$
1	-2	$-4e^{-1}$
-1	-2	$4e^{-1}$

Answer: $(x, y, z) =$

$(0,0,0)$, $(1, 2, 4e^{-1})$, $(-1, 2, -4e^{-1})$, $(1, -2, -4e^{-1})$ and $(-1, -2, 4e^{-1})$

12.5/13 Because the $+x^2$ and $+y^2$ are the dominant terms, this one opens upward.

$f_x = 2x - 2 \qquad f_y = 2y - 2.$

$f_y = f_x = 0$ when $x = 1$ and $y = 1$.

$z = 1^2 - 2 + 1^2 - 2 + 3 = 1$

The lowest point is $(x, y, z) = (1, 1, 1)$

Here's a way to do it with just algebra (no derivatives). Complete the squares:

$z = x^2 - 2x + y^2 - 2y + 3$

$= (x^2 - 2x + 1) + (y^2 - 2y + 1) + 1$

$= (x-1)^2 + (y-1)^2 + 1$

Since a square term is ≥ 0, the minimum z-value is 1 and it occurs when $x = 1$, $y = 1$.

~~12.5/22 z =~~

12.5/19 $z = 2x^2 + 8xy + y^4$. Opens upward.

$z_x = 4x + 8y \qquad z_y = 8x + 4y^3$

Set $z_x = 0$, $z_y = 0$

$\left.\begin{array}{l} x + 2y = 0 \\ 2x + y^3 = 0 \end{array}\right\}$ $x = -2y$. Substitute into 2^{nd} equation to get:

$2(-2y) + y^3 = 0 \qquad y(y^2 - 4) = 0$

$y(y-2)(y+2) = 0. \qquad y = 0, 2, -2$

Since $x = -2y$, the candidates for lowest point are $(x,y) = (0,0)$ $(-4,2)$ $(4,-2)$.

Find the z-values:

x	y	z
0	0	0
-4	2	$2(16) + 8(-4)(2) + 2^4 = -16$
4	-2	-16

There are two lowest points: $(-4,2,-16)$, $(4,-2,-16)$.
$(0,0,0)$ has a horizontal tangent plane, but it is not a minimum.

12.5/22 $z = (1+x^2)e^{-x^2-y^2}$

Note $z \geq 0$ for all (x, y).

As $x \to \infty$ and $y \to \infty$, $z \to 0$, since the exponential term $e^{-x^2-y^2}$ dominates the polynomial $1+x^2$. So the graph open downward and we look for a highest point.

$$z_x = 2x e^{-x^2-y^2} + (1+x^2)e^{-x^2-y^2}(-2x)$$

$$= e^{-x^2-y^2}(2x - 2x - 2x^3) = -2x^3 e^{-x^2-y^2}$$

$$z_y = (1+x^2)e^{-x^2-y^2}(-2y).$$

$z_x = 0$ only when $x = 0$.

$z_y = 0$ only when $y = 0$.

$z = (1+0)e^0 = 1.$

The max occurs at $(0, 0, 1) = (x, y, z)$.

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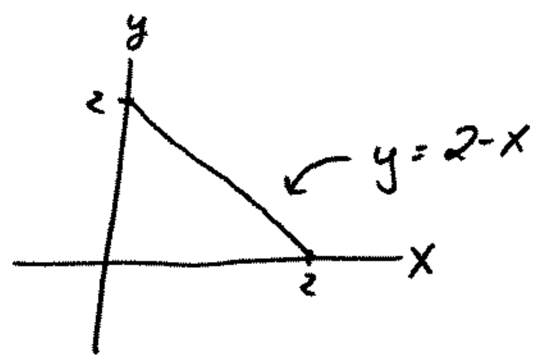

$f(x,y) = x^2 + y^2 - 2x$

$f_x = 2x-2$ $\quad f_y = 2y$

So $\boxed{(x,y) = (1,0)}$ is a critical point.

$f(1,0) = 1+0-2 = \boxed{-1}$

Now check the boundary pieces

On $y=0$, $0 \le x \le 2$, $\quad f(x,0) = x^2 - 2x$

$\frac{d}{dx}(x^2-2x) = 2x-2 = 0$ for $x=1$. $\quad (x,y) = (1,0)$

Check end points $(x,y) = (0,0), (2,0)$ also

On $x=0$ $\quad f(0,y) = y^2 \quad (0 \le y \le 2)$.

This is an increasing function for $0 \le y \le 2$.

Check end points $(x,y) = (0,0), (0,2)$

On $y = 2-x$, $f(x, 2-x) = x^2 + (2-x)^2 - 2x$

$= x^2 + 4 - 4x + x^2 - 2x = 2x^2 - 6x + 4$

$\frac{d}{dx}(2x^2-6x+4) = 4x-6 = 0$ for $x = 3/2$.

When $x = 3/2$, $y = 2-x = 1/2$. $\quad (3/2, 1/2)$.

x	1	0	2	0	3/2
y	0	0	0	2	1/2
$f(x,y)$	-1	0	0	4	-1/2

Max is 4, at $(x,y) = (0,2)$.

Min is -1, at $(x,y) = (1,0)$