

Solutions

Name: _____

Circle your Recitation Section: 8:00 10:00 12:00 2:00 3:00

Math 242, Section BL1 - Test #1 - October 3, 2005

Time: 55 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (6 points) Give the equation of the plane which is normal to $\mathbf{n} = \langle 9, 2, 1 \rangle$ and which contains the point $P = (0, -8, 3)$.

$$\vec{n} \cdot (\langle x, y, z \rangle - P) = 0$$

$$\langle 9, 2, 1 \rangle \cdot \langle x, y+8, z-3 \rangle = 0$$

$$\boxed{9x + 2(y+8) + z - 3 = 0}$$

2. (6 points) Give the parametric equations of the line which passes through $P = (-7, 4, 8)$ and is parallel to the vector $\mathbf{v} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.

$$\langle x(t), y(t), z(t) \rangle = \vec{r}(t) = P + t\vec{v}$$

$$= \langle -7, 4, 8 \rangle + \langle 5t, -2t, 3t \rangle$$

$$\boxed{\begin{aligned} x(t) &= -7 + 5t \\ y(t) &= 4 - 2t \\ z(t) &= 8 + 3t \end{aligned}}$$

3. (10 points) Suppose a falling object follows the path in space given by

$$x(t) = 2 \cos 3t, \quad y(t) = 2 \sin 3t, \quad z = 200 - 8t.$$

Find the distance traveled by the object from time $t = 0$ to time $t = 10$.

$$x'(t) = -6 \sin 3t$$

$$y'(t) = 6 \cos 3t$$

$$z'(t) = -8$$

distance traveled along the path =

$$\int_0^{10} \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt$$

$$= \int_0^{10} \sqrt{36 \sin^2 3t + 36 \cos^2 3t + 64} dt$$

$$= \int_0^{10} \sqrt{36 + 64} dt$$

$$= \int_0^{10} \sqrt{100} dt$$

$$= \int_0^{10} 10 dt = 10t \Big|_0^{10} = \boxed{100}$$

4. (6 points) A point has spherical coordinates $(\rho, \phi, \theta) = (6, \frac{\pi}{4}, \frac{\pi}{2})$. Find its rectangular coordinates (x, y, z) .

$$x = \rho \cos \theta \sin \phi = 6 \cos \frac{\pi}{2} \sin \frac{\pi}{4} = 6 \cdot 0 \cdot \frac{\sqrt{2}}{2} = 0$$

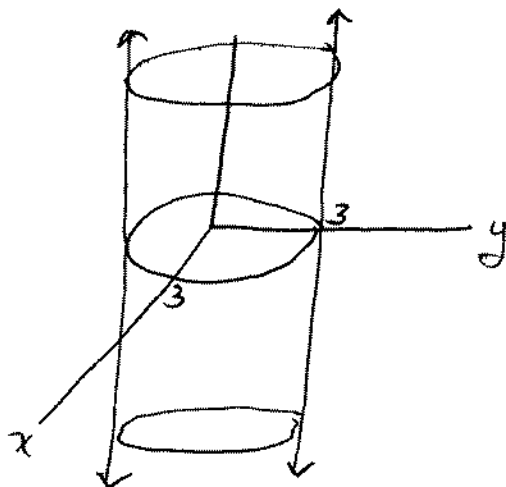
$$y = \rho \sin \theta \sin \phi = 6 \sin \frac{\pi}{2} \sin \frac{\pi}{4} = 6 \cdot 1 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

$$z = \rho \cos \phi = 6 \cos \frac{\pi}{4} = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

$$(x, y, z) = (0, 3\sqrt{2}, 3\sqrt{2})$$

5. (6 points) Describe and sketch the graph of the surface given in cylindrical coordinates by $r = 3$.

This is a circular cylinder of radius 3, centered on the z -axis.



6. (12 points) The symmetric equations of two lines are given below. These lines are parallel. Find a normal vector for the plane containing these two lines.

$$x - 1 = 2(y + 1) = 3(z - 2)$$

$$x - 3 = 2(y - 1) = 3(z + 1)$$

Rewrite in standard form: $\frac{x-1}{6} = \frac{y+1}{3} = \frac{z-2}{2}$

$$\frac{x-3}{6} = \frac{y-1}{3} = \frac{z+1}{2}$$

For both lines, the direction vector is

$\vec{u} = \langle 6, 3, 2 \rangle$. \vec{u} is parallel to the plane.

To find another vector parallel to the plane, observe that $P = (1, -1, 2)$ is a point on the 1st line and $Q = (3, 1, -1)$ on the second

Then $\vec{v} = \vec{PQ}$ is parallel to the plane.
 $= \langle 2, 2, -3 \rangle$

A normal vector to the plane is $\vec{n} = \vec{u} \times \vec{v}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = \vec{i} \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} 6 & 2 \\ 2 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} 6 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= -13\vec{i} + 22\vec{j} + 6\vec{k}$$

$$\vec{n} = -13\vec{i} + 22\vec{j} + 6\vec{k} \quad \text{or} \quad \langle -13, 22, 6 \rangle$$

7. (6 points) Define the dot product of two vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$. (Your definition can be a formula.)

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

8. (9 points) The position vector of a particle moving in space is given by $\mathbf{r}(t) = (2 \cos t)\mathbf{i} - 3t\mathbf{j} + (2 \sin t)\mathbf{k}$. Find its velocity and acceleration vectors and its speed.

velocity $\vec{v}(t) = (-2 \sin t)\vec{i} - 3\vec{j} + (2 \cos t)\vec{k}$

accel $\vec{a}(t) = (-2 \cos t)\vec{i} - (2 \sin t)\vec{k}$

speed $v(t) = |\vec{v}(t)| = \sqrt{4 \sin^2 t + 9 + 4 \cos^2 t}$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

9. (15 points) Describe and sketch the graph of the equation $9x^2 - 4y^2 - z^2 = 36$.

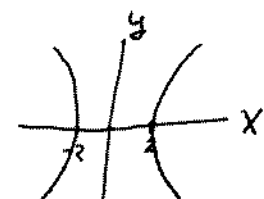
You do not have to give the name of the surface (although you may, to better describe it). You should

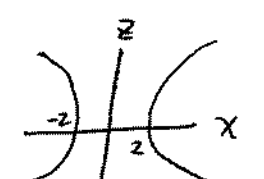
- Label the coordinate axes.
- Give the intercepts of the surface with each axis.
- Describe the intersections of the surface with the xy -plane, the xz -plane and the yz -plane.

x -intercepts Set $y=0, z=0$ $9x^2 = 36$ $x = \pm 2$

y -intercepts Set $x=0, z=0$ $-4y^2 = 36$ none

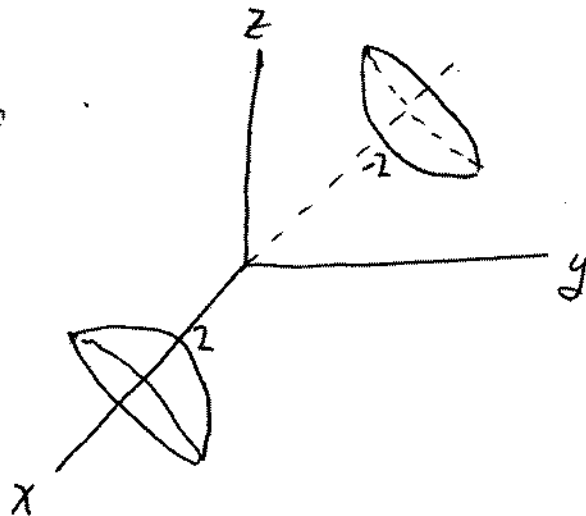
z -intercepts Set $x=0, y=0$ $-z^2 = 36$ none

intersection with xy plane:
 set $z=0$ $9x^2 - 4y^2 = 36$  hyperbola

intersection with xz -plane:
 set $y=0$ $9x^2 - z^2 = 36$  hyperbola

intersection with yz -plane:
 set $x=0$ $-4y^2 - z^2 = 36$ none

(a hyperboloid of two sheets)



10. (3 points each part) Answer *True* or *False* for each part. No explanation is needed and this question has no partial credit, just right or wrong.

(a) If θ is the angle between the vectors \mathbf{u} and \mathbf{v} , then $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\sin\theta$.

F $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

(b) The vectors $8\mathbf{i} + 6\mathbf{j}$ and $-3\mathbf{i} + 4\mathbf{j}$ are perpendicular.

T because their dot product is 0.

(c) If $\mathbf{r}(t)$ is the position vector of a curve and if $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are nonzero, then the unit tangent vector of the curve at the point $\mathbf{r}(t)$ is

F should be $\frac{\vec{r}'(t)}{|\vec{r}'(t)|}$

(d) The cross product $\mathbf{a} \times \mathbf{b}$ of nonzero vectors \mathbf{a} and \mathbf{b} is $\mathbf{0}$ if and only if \mathbf{a} and \mathbf{b} are parallel.

T See p. 793

(e) For any vectors \mathbf{a} and \mathbf{b} , $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$.

T See p. 794

(f) If \mathbf{a} , \mathbf{b} , \mathbf{c} are space vectors with the same initial point, so that they form the three sides of a parallelepiped, then the volume of the parallelepiped is $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

T See p. 795

(g) If a particle moving in space has constant speed $v(t)$, then its acceleration vector $\mathbf{a}(t)$ is $\mathbf{0}$.

F For example $\vec{r}(t) = \langle \cos t, \sin t \rangle$ has speed $v(t) = 1$
and $\vec{a}(t) = \langle -\cos t, -\sin t \rangle \neq \vec{0}$.

(h) The curvature κ of a straight line in space is zero.

T See p. 821