

Name: \_\_\_\_\_

## Math 247, Section F1 - Test #2 - March 24, 2000

Note: the points add to more than 100 because I have combined problems from several exams that I gave in past semesters for this practice exam Fall 2002.

- (15 points) Prove that if  $g : A \rightarrow B$  is an injection and  $f : B \rightarrow C$  is an injection, then  $f \circ g$  is an injection.
- (15 points) Out of 100 senators, an ethics committee of 5 will be chosen, with one of the committee members designated as chairperson. How many ways are there to do this?
- Let  $d = \gcd(5891, 6987)$ .
  - (10 points) Find  $d$ .
  - (10 points) Find integers  $m$  and  $n$  such that  $5891m + 6987n = d$ .
- (15 points) Prove that for all  $a, b \in \mathbf{Z}$ ,  $\gcd(a, b) = \gcd(a - 2b, b)$ . (There is a more general theorem which implies this, but I am asking for a proof in this specific case.)
- (15 points) Let  $A \subseteq \mathbf{R}$ ,  $B \subseteq \mathbf{R}$  and let  $f : A \rightarrow B$  be a bijection. Prove that if  $f$  is increasing on  $A$ , then  $f^{-1}$  is increasing on  $B$ .
- (20 points) Prove that  $\sqrt{13}$  is irrational.

*Note: There is a theorem which says "The positive integer  $k$  has no rational square root if  $k$  is not the square of an integer." If you choose to use this theorem in your proof, you must give a proof of the theorem first.*
- (10 points) Suppose that  $x$  is irrational,  $a$  is rational and  $a \neq 0$ . Prove that  $ax + a$  must be irrational.
- (10 points) Give an example to show that the statement "If  $x$  is irrational and  $y$  is irrational, then  $xy$  must be irrational." is *false*.