Name:____

Math 247, Section F1 - Test #3 - April 17, 2000

Time: 50 minutes. You may not use any books or notes. A calculator is permitted. Partial credit will be given based on what it written, not what is intended. There are 100 points total.

1. (20 points) Prove that $\sqrt{13}$ is irrational.
   Note: There is a theorem which says "The positive integer k has no rational square root if k is not the square of an integer." If you choose to use this theorem in your proof, you must give a proof of the theorem first.

2. (20 points) Use the Inclusion-Exclusion Principle to determine how many natural numbers are less than 338 and relatively prime to 338. ($338 = 2 \cdot 13 \cdot 13$.)

3. (a) (10 points) Suppose that x is irrational, a is rational and $a \neq 0$. Prove that $ax + a$ must be irrational.

(b) (10 points) Give an example to show that the statement "If x is irrational and y is irrational, then xy must be irrational." is *false*.

4. (a) (10 points) State the definition of "The sequence $< a_n >$ of real numbers converges to the limit $L \in \mathbf{R}$."

(b) (10 points)Use the definition from part (a) to prove that the sequence $< \frac{1}{n^3} >$ converges to 0.

5. (20 points) Suppose $< a_n >$ is a sequence of real numbers which converges to $A \in \mathbf{R}$ and $< b_n >$ is a sequence of real numbers which converges to $B \in \mathbf{R}$. Prove that if $A < B$, then there exists a natural number N such that $n \geq N$ implies $a_n < b_n$.