

## Math 315, Section C2, Fall 2002, December 16, 2002

Some Review Problems for material covered since Exam 3.

Note: This is just a sampling of problems. It does not include everything. Assigned homework problems provide further review problems.

1. Give the definitions of *orthogonal matrix*, *eigenvalue*, *eigenvector*, *eigenspace*, *diagonalizable matrix*, *singular value decomposition*
2. Prove that if  $Q$  is an orthogonal matrix, then (with the usual inner product on  $\mathbf{R}^n$ )

$$\langle Q\mathbf{x}, Q\mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle .$$

3. Use the Gram-Schmidt process to find an orthonormal basis for the subspace of  $\mathbf{R}^4$  spanned by  $(1, 2, -2, 0)^T$ ,  $(4, 3, 2, 0)^T$ ,  $(1, 2, 1, 0)^T$ .
4. Explain how the  $QR$  factorization can help when solving a least squares problem (see 5c from Section 5.6).
5. Be able to find the eigenvalues and corresponding eigenspaces for a square matrix. There are many practice problems in Section 6.1, exercise 1.
6. Give an example of a  $2 \times 2$  matrix which has no real eigenvalues.
7. Prove that if  $\lambda$  is an eigenvalue of  $A$ , then  $\lambda^2$  is an eigenvalue of  $A^2 = AA$ .
8. Find a diagonalization  $XDX^{-1}$  for a matrix  $A$ , if there is one. Exercise 1 from Section 6.3 gives you several matrices to practice on.
9. Show that if  $A$  and  $B$  are two  $n \times n$  matrices that both have the same diagonalizing matrix  $X$ , then  $AB = BA$ . Hint: The diagonal matrix  $D$  will be different for  $A$  and for  $B$ . Do diagonal matrices commute with each other?
10. Find the singular value decomposition of the matrix

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 0 & 0 \end{bmatrix}$$