Math 315, Section C2, Fall 2002, September 20, 2002

Some True/False Review Problems for Chapters 1 and 2

Note: This is just a sampling of problems. It does not include everything. For each problem, decide whether or not it is true for all such matrices. Give a proof or explain your reasoning if true; give a counterexample if false.

1. If the row echelon form of A involves free variables, then the system $A\mathbf{x} = \mathbf{b}$ will have infinitely many solutions.
2. A homogeneous linear system always has a solution.
3. An $n \times n$ matrix A is nonsingular if and only if the reduced row echelon form of A is I.
4. If A and B are nonsingular $n \times n$ matrices, then $A + B$ is also nonsingular and $(A + B)^{-1} = A^{-1} + B^{-1}$.
5. If A and B are nonsingular $n\times n$ matrices, then AB is also nonsingular and $(AB)^{-1} = A^{-1}B^{-1}$.
6. If $AB = AC$ and $A \neq 0$ (the zero matrix), then $B = C$.
7. The product of two elementary matrices is an elementary matrix.
8. Let A be a 4×3 matrix with $\mathbf{a}_2 = \mathbf{a}_3$. If $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$, then the system $A\mathbf{x} = \mathbf{b}$ will have a solution.
9. Let A be a 4×3 matrix with $\mathbf{a}_2 = \mathbf{a}_3$. If $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$, then the system $A\mathbf{x} = \mathbf{b}$ will have infinitely many solutions.
10. $\det(AB) = \det(BA)$
11. $\det(A + B) = \det(A) + \det(B)$.
12. $\det(cA) = c\det(A)$
13. $\det((AB)^T) = \det(A)\det(B)$.
14. $\det(A) = \det(B)$ implies $A = B$.
15. A triangular matrix is nonsingular if and only if its diagonal entries are all nonzero.
16. If $\mathbf{x}$ is a nonzero vector in R^n and $A\mathbf{x} = \mathbf{0}$, then $\det(A) = 0$.
17. If A and B are row equivalent to one another, then their determinants are equal.
18. If A is nonsingular, then $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.
19. Every elementary matrix is nonsingular.
20. $(AB)^T = A^TB^T$.
21. If $A\mathbf{x} = \mathbf{b}$, then $\mathbf{b}$ is a linear combination of the columns of A.
22. If $\mathbf{b}$ is a linear combination of the columns of A, then $A\mathbf{x} = \mathbf{b}$ is consistent.