Math 315, Section C2, Fall 2002, September 20, 2002

Solutions - Some True/False Review Problems for Chapters 1 and 2

Note: This is just a sampling of problems. It does not include everything. For each problem, decide whether or not it is true for all such matrices. Give a proof or explain your reasoning if true; give a counterexample if false.

1. False, because it could be that there is no solution. See Example 5a on page 18.

2. True. $\mathbf{x} = \mathbf{0}$ is solution.

3. True. By Theorem 1.4.3 on page 71-72, A is nonsingular if and only if it is row equivalent to I, which means its reduced row echelon form is I.

4. False. For example, if $A = I$ and $B = -I$, then A and B are nonsingular but $A + B$ is the zero matrix, which is singular.

5. False. It is true that AB is nonsingular, but the correct identity is $(AB)^{-1} = B^{-1}A^{-1}$.

6. False. The statement would be true if A were nonsingular, but A might be singular, even if it is not the zero matrix. For example, let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}.$$

7. False. For example,
$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}.$$

8. True. $\mathbf{x} = [1\ 1\ 1]^T$ is a solution. The condition $\mathbf{a}_2 = \mathbf{a}_3$ is irrelevant for this question.

9. True. Since $\mathbf{a}_2 = \mathbf{a}_3$, $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 = \mathbf{a}_1 + 2\mathbf{a}_2 + 0\mathbf{a}_3$. Therefore $[1\ 2\ 0]^T$ is another solution in addition to the one mentioned in the previous question. This shows that $A\mathbf{x} = \mathbf{b}$ has at least two solutions, so it has an infinite number of solutions.

10. True. By the theorem on page 112, $\det(AB) = \det A \det B$. Using the same theorem, $\det(BA) = \det B \det A$. Using just the commutative property of numbers, we conclude that $\det(AB) = \det(BA)$.

11. False. For example, let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$
Then $\det A = \det B = 0$ but $\det(A + B) = 1$.

12. False. The correct statement is $\det(cA) = c^n \det A$, where n is the number of rows of A.

13. True. By a theorem on page 105, $\det((AB)^T) = \det(AB)$, and this equals $\det A \det B$ by the theorem on page 112.

14. False. For example, let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}.$$

15. True. A matrix is nonsingular if and only if its determinant is nonzero. The determinant of a triangular matrix is the product of the diagonal entries (see p. 105), which will be nonzero if and only if each diagonal entry is nonzero.

16. True. By the theorem on pages 71-72, $A\mathbf{x} = \mathbf{0}$ has a nontrivial (nonzero) solution $\mathbf{x}$ if and only if A is nonsingular, and by the theorem on page 110, A is nonsingular if and only if $\det A = 0$.

17. False. See pages 108-109 for information on how the determinant changes when a row operation is done.

18. False. By the theorem on pages 71-72, if A is nonsingular, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

19. True. See Section 4 of Chapter 1.

20. False. The correct identity is $(AB)^T = B^T A^T$.

21. True. This follows from the definition of matrix multiplication. See page 41.

22. True. If $\mathbf{b}$ is a linear combination of the columns of A, this means there are scalars $x_1, \ldots x_n$ such that

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b},$$

where the $\mathbf{a}_i$ are the columns of A, so $\mathbf{x} = [x_1\ x_2\ \ \ldots x_n]^T$ is a solution of $A\mathbf{x} = \mathbf{b}$, so the system is consistent.