Math 344, Section X1, Fall 2001

Review Problems for Chapters 1 and 2

Note: This is just a sampling of problems. It does not include everything.

1. Section 1.1

   (a) Give the definitions of "$f : A \to B$ is injective" and "$f : A \to B$ is surjective."

   (b) Give an example of a function which is injective and surjective, a function which is injective but not surjective, a function which is surjective but not injective, and a function which is neither injective nor surjective.

   (c) Prove that if $f : A \to B$ and $g : B \to C$ and if $g \circ f$ is injective, then f must be injective.

2. Section 1.3

   (a) Give the definitions of "denumerable set", "countable set" and "uncountable set".

   (b) Give an example of each type of set from part (a).

   (c) Prove that the set of all vectors $\mathbf{v} = (v_1, v_2)$, where v_1 and v_2 are integers, is countable.

3. Section 2.2

   (a) Give the definition of $|a|$.

   (b) Prove that for all $a, b \in \mathbf{R}$, $|a + b| \leq |a| + |b|$.

   (c) Give an example where $|a + b| < |a| + |b|$. Give an example where $|a + b| = |a| + |b|$.

4. Section 2.3

   (a) Give the definitions of upper bound of S, lower bound of S, $\sup S$, $\inf S$.

   (b) Give the statement of the Completeness Property of **R**.

   (c) Give an example of a set which has an upper bound but no lower bound.

   (d) Give an example of a bounded set S such that $\sup S$ is not an element of S.

   (e) Prove that if $A \subseteq B$ and A and B are both bounded, then $\inf A \geq \inf B$.

5. Section 2.4

   (a) Define "rational number" and "irrational number".

   (b) Prove that if x and y are both rational, then $x + y$ is rational.

   (c) Prove that if x is rational and $x \neq 0$ and y is irrational, then xy is irrational.

   (d) Prove that if $x < y$ are two real numbers, then there is an irrational number z such that $x < z < y$ (i.e. fill in the details of the proof of Corollary 2.4.9.)