Math 344, Section X1, Fall 2001

Review Problems for Chapters 6 and 7 and Section 5.4

Note: This is just a sampling of problems. It does not include everything.

1. Section 6.1
   (a) Give the definitions of "f is differentiable at c" and "f is differentiable on the set I".
   (b) Give an example of a function which is differentiable on $\mathbf{R}$, a function which is continuous on $\mathbf{R}$ but which is not differentiable at 0, a function which is defined on $\mathbf{R}$ but which is not differentiable at any point.
   (c) Prove that if f and g are both differentiable at c, then fg is differentiable at c and $(fg)' = f'g + fg'$.
   (d) State the Chain Rule, including all hypotheses.
2. Section 6.2
   (a) State the Interior Extremum Theorem and give a proof in the case where f has a relative minimum at c.
   (b) Is the Interior Extremum Theorem true if c is an endpoint of I? Give a counterexample or a proof.
   (c) State the Mean Value Theorem.
   (d) Is the Mean Value Theorem true if the hypothesis of continuity on $[a, b]$ is dropped? Prove or give a counterexample.
   (e) Is the Mean Value Theorem true if the hypothesis of diffferentiability on (a, b) is dropped? Prove or give a counterexample.
3. Section 7.1
   (a) Give the definition of Riemann integrable.
   (b) Prove, directly from the definition, that a constant function is Riemann integrable on the interval $[a, b]$.
   (c) Is it true that a bounded function must be integrable on $[a, b]$? Give some examples.
   (d) Prove (from the definition of Riemann integral), that if f is Riemann integrable on $[a, b]$, then $\int_a^b 2f = 2\int_a^b f$.
4. Section 7.2
   (a) State the Cauchy Criterion and give an example of how it can be used.
   (b) State the Squeeze Theorem and give an example of how it can be used.
   (c) True or false (give examples or cite Theorems from the book to support your answer.):

i. Every continuous function on $[a, b]$ is Riemann integrable.
ii. Every step function on $[a, b]$ is Riemann integrable.
iii. Every bounded function on $[a, b]$ is Riemann integrable.
iv. Every monotone function on $[a, b]$ is Riemann integrable.
v. Every function which is Riemann integrable on $[a, b]$ must be continuous.
vi. Every function which is Riemann integrable on $[a, b]$ must be a step function
vii. Every function which is Riemann integrable on $[a, b]$ must be bounded.
viii. Every function which is Riemann integrable on $[a, b]$ must be monotone.

5. Section 7.3

   (a) State the two forms of the Fundamental Theorem of Calculus. Be sure to include all hypotheses.

   (b) Find $F'(x)$ when F is defined on $[0, 1]$ as follows (verify that the hypotheses of whatever theorem you use are satisfied):

$$F(x) = \int_{x^3}^{x} \sin(t^2 + 1)\, dt.$$

6. Section 5.4

   (a) Give the definition of "f is uniformly continuous on $A \subset \mathbf{R}$.

   (b) Give an example of a function which **is** uniformly continuous on the open interval $(0, 1)$.

   (c) Give an example of a function which **is not** uniformly continuous on the open interval $(0, 1)$.

   (d) Prove that if (x_n) is a Cauchy sequence in a set $A \subset \mathbf{R}$ and if f is uniformly continuous on A, then $(f(x_n))$ is a Cauchy sequence.