Math 344, Section X1, Fall 2001

Review Problems for Section 5.1-5.3, 8.1

Note: This is just a sampling of problems. It does not include everything.

1. Section 5.1

   (a) Give the definitions of "f is continuous at c" and "f is continuous on the set I".

   (b) Give an example of a function which is continuous on $\mathbf{R}$, a function which is continuous on $\mathbf{R} \setminus \{0\}$ but which is not continuous at 0, a function which is defined on $\mathbf{R}$ but which is not continuous at any point.

   (c) State the Sequential Criterion for Continuity.

   (d) Use the Sequential Criterion to prove that the f is continuous at 0 but, for all $c \neq 0$, is not continuous:

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

2. Section 5.2

   (a) Prove that if f and g are both continuous at c, then fg is continuous at c.

   (b) Prove that the composition of continuous functions is continuous.

3. Section 5.3

   (a) True or False?: A continuous function on the interval $[2, 3]$ must be bounded.

   (b) True or False?: If f is continuous on the interval $(0, 2)$, then f must have an absolute maximum on $(0, 2)$.

   (c) State the Intermediate Value Theorem (p. 133)

   (d) Use the Intermediate Value Theorem to prove if f is continuous on $\mathbf{R}$ and if $f(x)$ is irrational for all x, then f must be constant.

4. Section 8.1

   (a) Explain the difference between pointwise convergence of a sequence of functions and uniform convergence of a sequence of functions. Give the precise definition of each.

   (b) Give an example of a sequence of continuous functions which converges pointwise to a non-continuous function on some interval.

   (c) Give an example of a sequence of functions which converges uniformly on $\mathbf{R}$.

   (d) True or false?: If a sequence of continuous functions converges uniformly on A to a function f, then f must be continuous on A. (See 8.2.2 on p. 234).