Math 385

Chapter 3 Practice Problems (does not include everything!)

Part I Solving equations

1. Homogeneous, Constant Coeff., Linear

   a. $y^{(4)} - 9y''' + 29y'' - 39y' + 18y = 0$

   hint: characteristic polynomial factors as $(r^2 - 3r + 2)(r^2 - 6r + 9)$

   b. $y'' + 4y = 0$

   c. $y^{(4)} + 2y'' + y = 0$

   d. $y'' - 6y' + 13y = 0$

2. Non-homogeneous, constant coeff., linear (Undetermined Coefficients) Find general solution

   a. $y'' + 4y = 3x + e^x$

   b. $y'' + 4y = \cos 2x$

   c. $y'' - 6y' + 13y = x e^{3x} \sin 2x$

3. Variation of Parameters.
(if the integrals are hard to evaluate, just set them up + don't evaluate).

a. $y'' + 4y = \ln x \qquad (x > 0)$

b. $y'' - 6y' + 13y = \sqrt{x^2+1}$

4. End point problems

a. Solve $y'' + 2y' + (4\pi^2 + 1)y = 0$, $y(0) = y(1) = 0$

b. Find the eigenvalues and eigenfunctions

$$y'' + \lambda y = 0 \; ; \; y(-2) = 0, \; y'(2) = 0$$

Part II Applications

5. Set up spring problem.

A spring is stretched 2 m by a force of 96 N/m. A .75 kg mass is attached and it is set in motion with initial position $X(0) = 1$ and initial velocity $X'(0) = 0$.

Set up the equation and find the solution (position function).

6. Same as problem 5, but with a dashpot with damping constant ~~2~~6.

7. Same as 5., with a forcing function $\cos(2t)$.

8. Same as 6., with a forcing function $\cos(2t)$.

9. In 5., what forcing function $\cos(\omega t)$ will result in pure resonance?

10. In 5., what forcing function $\cos(\omega t)$ will result in beats (resonance, not pure resonance)

11. In 6., what forcing function $\cos(\omega t)$ (if any), will result in practical resonance?

12. For 8., find and graph x_{tr} and x_{sp}

13. Sketch graphs for position functions in problems 5-11. Use calculator if needed. First - approx. what shape do you expect?