Name:____

Math 385, Section D1 - Test #2- November 4, 2005

Time: 50 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (14 points) Find the solution to the initial value problem

$$y'' - 4y' + 4y = 0, \qquad y(0) = 0,\ y'(0) = 3.$$

characteristic equation $r^2 - 4r + 4 = 0$

$(r-2)^2 = 0 \qquad r = 2, 2$

$y(x) = c_1 e^{2x} + c_2 x e^{2x}$

$0 = y(0) = c_1$ so $y(x) = c_2 x e^{2x}$

$y'(x) = c_2 e^{2x} + 2c_2 x e^{2x}$

$3 = y'(0) = c_2$

Solution: $y(x) = 3x e^{2x}$

2. (12 points) This problem tests your knowledge of how to set up "Undetermined Coefficients." Set up the appropriate form of a particular solution y_p, then stop there - do not plug in and do not determine the values of the coefficients.

$$y'' + 16y = x^2 + \sin 4x$$

characteristic equation: $r^2 + 16 = 0$

$r = \pm 4i$ $\quad\quad y_c = c_1 \cos 4x + c_2 \sin 4x$

$y_p = Ax^2 + Bx + C + Dx\cos 4x + Ex\sin 4x$

3. (12 points) Use **variation of parameters** to find a particular solution of

$$y'' - 4y = \ln x.$$

Put your answer in the form $y_p(x) =$ ______, but do **not** try to evaluate the integrals - just leave them as integrals in your answer. You are given the formulas

$$u_1 = -\int \frac{y_2 f}{W}\,dx, \qquad u_2 = \int \frac{y_1 f}{W}\,dx.$$

characteristic equation $r^2 - 4 = 0$

$(r-2)(r+2) = 0 \qquad r = 2, -2$

$y_1(x) = e^{2x} \qquad y_2(x) = e^{-2x}$

$y_1'(x) = 2e^{2x} \qquad y_2'(x) = -2e^{-2x}$

$W = y_1 y_2' - y_2 y_1' = -2 - 2 = -4$

$f(x) = \ln x$

$y_p = u_1 y_1 + u_2 y_2 =$

$-e^{2x} \int \frac{e^{-2x} \ln x}{-4}\,dx + e^{-2x} \int \frac{e^{2x} \ln x}{-4}\,dx$

4. (12 points) For the eigenvalue problem below, show that $\lambda = 0$ is an eigenvalue and find the associated eigenfunction. Note: you are **not** being asked to find all the eigenvalues! Just show that $\lambda = 0$ is an eigenvalue.

$$y'' + \lambda y = 0 \qquad y(0) = 0, \; y(1) = y'(1).$$

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5. Consider the mass-spring system modelled by

$$x'' + 3x' + 2x = 0 \qquad x(0) = 1,\ x'(0) = 0.$$

Note: you are not asked for a solution function - just do as much work as needed to answer the following questions.

(a) (6 points) Is the system overdamped, critically damped, or underdamped? Explain briefly how you know (about 1 sentence).

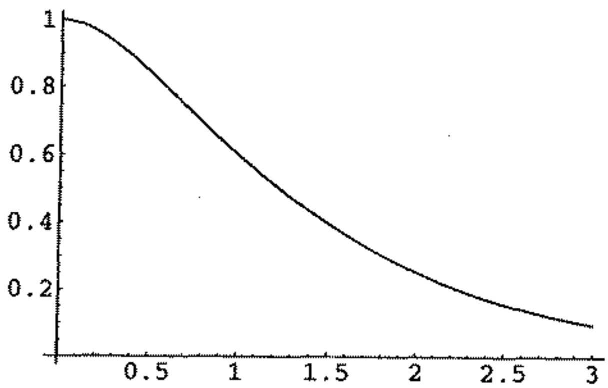
See Version 1

(b) (6 points) Which of the graphs on the following page is the graph of the position function $x(t)$? Explain briefly how you know (about 1 sentence).

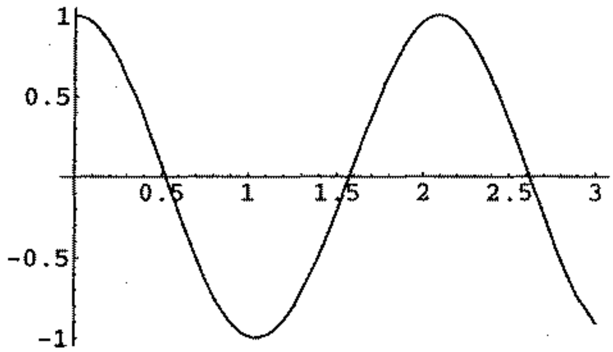
A. Decay without oscillation corresponds to overdamping.

Graphs for Question 5

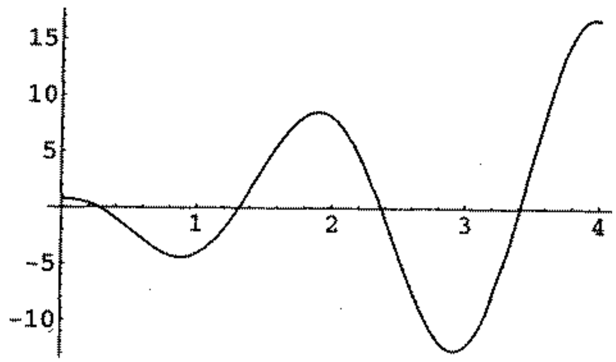
A



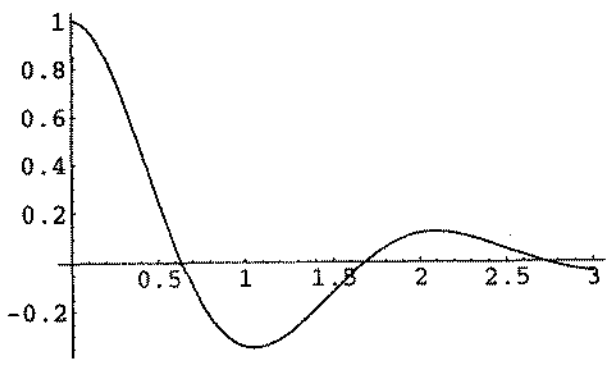
B



C



D



6. Consider the mass-spring system with forcing function modelled by

$$x'' + 9x = \cos(\omega t) \qquad x(0) = 0,\ x'(0) = 0.$$

Note: you are not being asked to find a solution function.

(a) (6 points) What frequency ω in the forcing function will result in pure resonance? Explain briefly (1 sentence).

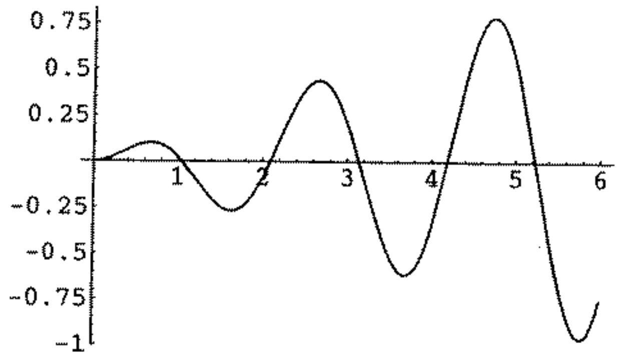
See version 1

(b) (6 points) Which of the graphs on the following page is the graph of the position function $x(t)$ showing pure resonance? Explain briefly (1 sentence).

A. See Version 1

Graphs for Question 6

A



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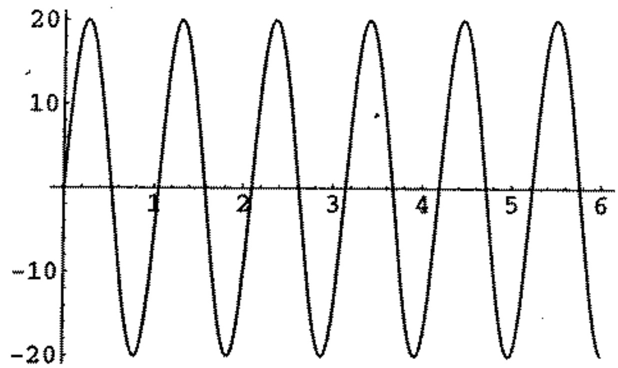

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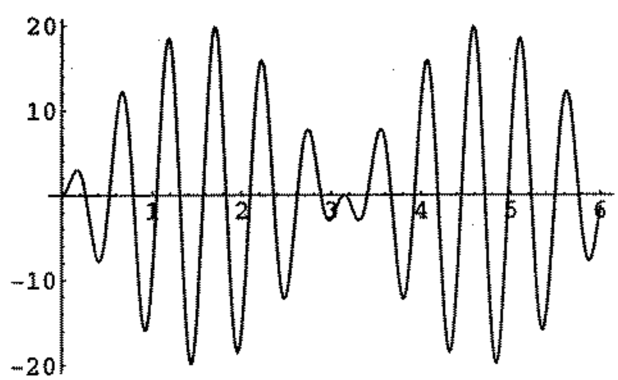
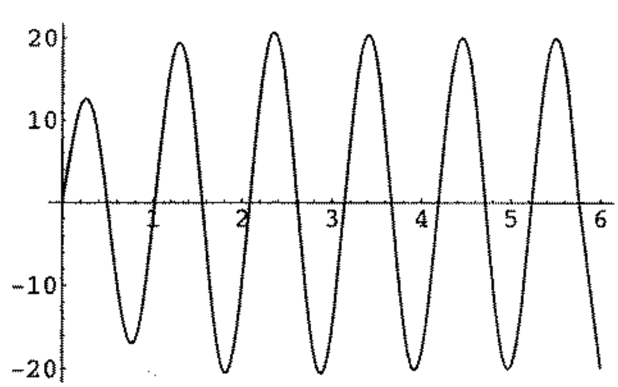


D



7. (13 points) Prove that if $Y_1(x)$ and $Y_2(x)$ are both solutions of

$$(*) \qquad y'' + p(x)y' + y = f(x),$$

then $Y_1(x) - Y_2(x)$ is a solution of

$$(**) \qquad y'' + p(x)y' + y = 0.$$

See Version 1.

8. (a) (7 points) Define what it means for two functions $f_1(x)$ and $f_2(x)$ to be linearly independent on the interval I.

See Version 1.

(b) (6 points) Explain how the Wronskian is used in determining whether two solutions y_1, y_2 of a 2nd order homogeneous linear differential equation are linearly independent or linearly dependent.

See Version 1.