Name:____

Math 385, Section D1 - Test #1 - September 30, 2005

Time: 55 minutes. You may not use any books or notes or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (5 points each part) Classify each of the following differential equations as linear, separable, exact or homogeneous. No explanation of your answer is needed. Do not solve the differential equations.

   (a)
   $$\frac{dy}{dx} = \frac{e^{-y}}{(x+1)(x-2)}$$

   (b)
   $$\frac{dy}{dx} = \frac{x + \sin y}{2y - x\cos y}$$

   (c)
   $$\frac{dy}{dx} = \frac{2y^2 + x^2}{xy}$$

   (d)
   $$\frac{dy}{dx} = x^4 - 4x^{-1}y$$

2. (24 points) Find a general solution for any 2 of the differential equations from question 1 (your choice). Be sure to clearly indicate which equations you are solving. You do not have to solve algebraically for y at the end; y can be given implicitly as a function of x.

3. (a) (5 points) Define what is meant by the *critical points* of an autonomous differential equation $\frac{dx}{dt} = f(x)$.

(b) (4 points) Find the critical points of the equation

$$\frac{dx}{dt} = 9 - x^2.$$

(c) (6 points) Determine whether each critical point from part (b) is stable or unstable.

4. Consider the initial value problem

$$\frac{dx}{dt} = x - t, \qquad x(1) = 0.$$

(a) (6 points) On the tx-plane, with t ranging from 0 to 2 and x ranging from 0 to 2, make a rough sketch of the slope field, drawing the line segments at points with integer coordinates only.

(b) (6 points) With step size $h = 1$, do the first step of the Euler method (to find the approximation x_1).

(c) (10 points) With step size $h = 1$, do the entire first step of the improved Euler method (to find the approximation x_1).

5. A tank contains 400 L of a solution consisting of 20 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5 L/sec, and the mixture - kept uniform by stirring - is pumped out at the same rate.

(a) (5 points) Set up a differential equation *and* initial condition for the kg of salt $x(t)$ in the tank after t seconds. (Do not solve the equation.)

(b) (5 points) The general solution for the differential equation in part (a) is $x(t) = Ke^{-t/200}$ Use this solution to determine how long will it be until only 10 kg of salt remains in the tank. Any exponentials or logarithms appearing in your answer do not have to be evaluated.

6. (3 points each part) Answer *True* or *False* for each part. No explanation is needed and this question has no partial credit, just right or wrong.

(a) An *equilibrium solution* means a constant solution of a differential equation.

(b) Given any function $f(x, y)$ and any initial condition $y(a) = b$, the differential equation

$$\frac{dy}{dx} = f(x, y)$$

has a unique solution with $y(a) = b$.

(c) The function $y(x) = x$ is a solution of the differential equation

$$\frac{dy}{dx} = \sin^2(x^2) + cos^2(xy).$$