Section 1.3

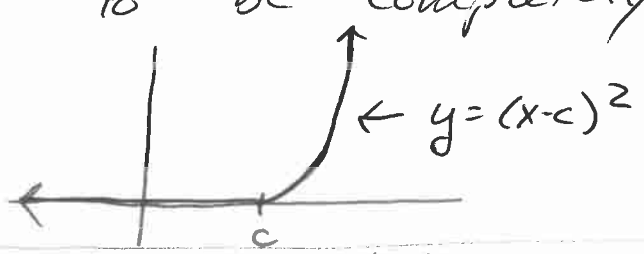
(27) a) For $x < c$, $y(x) \equiv 0$, so both the left and right hand sides of $y' = 2\sqrt{y}$ are identically 0.

For $x > c$, $y(x) = (x-c)^2$, so $y'(x) = 2(x-c)$ and $2\sqrt{y} = 2\sqrt{(x-c)^2} = 2(x-c)$. So $y' = 2\sqrt{y}$

For $x = c$, $2\sqrt{y} = 0$ since $y(c) = 0$.

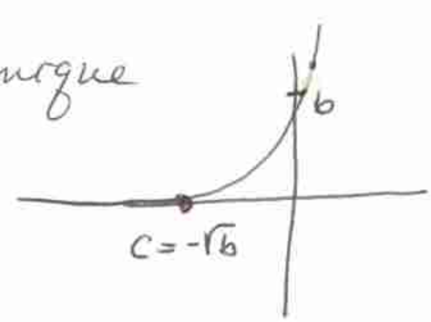
$y'(c) = 0$ also, since $\frac{d}{dx}\Big|_{x=c} 0 = 0$ and $\frac{d}{dx}\Big|_{x=c} (x-c)^2 = 0$. What I am trying to say is that approaching from both the left side and the right side, $y'(c) = 0$. (Use limit definition of y' to be completely rigorous)

The graph helps:



(b) If $b < 0$, there is no solution. The right hand side, $2\sqrt{y}$, isn't even defined when $y(0) = b < 0$.

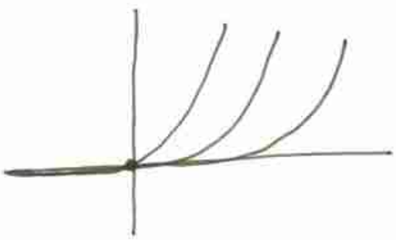
If $b > 0$, there is a unique solution:



cont'd →

27 b cont'd (Section 1.3)

If $b=0$, then there are infinitely many solutions.



Here are the graphs of three solutions having $y(0)=0$.

Section 1.4

8 $\frac{dy}{dx} = 2x \sec y$ $\qquad \frac{dy}{dx} \cos y = 2x$

$\int \cos y \, dy = \int 2x \, dx \qquad \sin y = x^2 + C$

$$\boxed{y = \sin^{-1}(x^2+C)}$$

16 $(x^2+1)(\tan y)\, y' = x$

$\int \tan y \cdot dy = \int \frac{x}{x^2+1} \, dx$

$-\ln|\cos y| = \frac{1}{2}\ln(x^2+1) + C$

$|\cos y|^{-1} = (x^2+1)^{\frac{1}{2}} e^C$

$\cos y = (x^2+1)^{-\frac{1}{2}} e^{-C}$

$$\boxed{y = \cos^{-1}\left(K(x^2+1)^{-\frac{1}{2}}\right)}$$

Section 1.4

(17) $\frac{dy}{dx} = 1 + x + y + xy = (1+x)(1+y)$

$\frac{1}{1+y}\,dy = (1+x)\,dx$ $\qquad \ln|1+y| = x + \frac{x^2}{2} + C$

$|1+y| = e^C e^{x+\frac{x^2}{2}}$ $\qquad 1+y = \pm e^C e^{x+\frac{x^2}{2}}$

$\boxed{y = K e^{x+\frac{x^2}{2}} - 1}$

(28) $2\sqrt{x}\,\frac{dy}{dx} = \cos^2 y$ $\qquad y(4) = \pi/4$

$2\int \sec^2 y\,dy = \int x^{-1/2}\,dx$

$2\tan y = 2x^{1/2} + C$

$\boxed{y = \arctan(x^{1/2} + K)}$, any K

(35) Let N = quantity of ^{14}C. Let t = time in years.

From page 36,

$\frac{dN}{dt} = -kN$, with $k = .0001216$

$\int \frac{1}{N}\,dN = \int -k\,dt$

$\ln|N| = -kt + C$

$N = Ce^{-kt}$

→

Section 1.4

35 cont'd.

Let $t=0$ be current time, $t = t_s$ = time of the skull (t_s is negative).

We are given $N(0) = \frac{1}{6} N(t_s)$

so $C e^{-k \cdot 0} = \frac{1}{6} C e^{-k t_s}$

This yields $6 = e^{-k t_s}$ $\qquad \ln 6 = -k t_s$

$t_s = \frac{\ln 6}{-k} \approx \boxed{14,735}$ years.

46 $\frac{dp}{dx} = -.2p$; $p(0) = 29.92$

Solve D.E. : $\int \frac{1}{p}\, dp = \int -.2\, dx$ $\qquad \ln|p| = -.2x + c$

(a) $\boxed{p = c e^{-.2x}}$ $\qquad$ When $x = 10,000$ ft. $= 1.9$ miles,

$p = 29.92\, e^{-(.2)(1.9)} = \boxed{20.46 \text{ inches of mercury}}$

when $x = 30,000$ ft. $= 5.7$ miles,

$p = 29.92\, e^{-(.2)(5.7)} = \boxed{9.57 \text{ inches of mercury}}$

(b) Set $p = 15$ $\qquad 15 = 29.92\, e^{-.2x}$ $\qquad$ Solve for x

$\frac{15}{29.92} = e^{-.2x}$ $\qquad -5 \ln\left(\frac{15}{29.92}\right) = x$

$\boxed{x = 3.45 \text{ miles} \approx 18,000 \text{ feet}}$

Iode Project #1

8. True. If $y(x)$ is periodic, then its slope is also periodic. The slope is given by dy/dx, which is called $f(x)$ in this problem.

9. False. It is possible for the slope $\frac{dy}{dx}$ to be periodic while the function $y(x)$ is not periodic. An example is
$y(x) = x + \sin x$, $\frac{dy}{dx} = 1 + \cos x$.

10. ... if $f(x,y)$ is periodic in y. In this case, as y changes, $\frac{dy}{dx} = f(x,y)$ will also change, but it will repeat itself because $f(x,y)$ is periodic in y.

11. False. An example is $y(x) = x$; $\frac{dy}{dx} = 1$.
$y(x) \to \infty$ as $x \to \infty$, but $f(x)$ is always 1.

12. The direction field will be horizontal — at height 2 because when $y=2$, $\frac{dy}{dx} = f(2) = 0$.

13. $\frac{dy}{dx} = 3 - y$ is one example that works.