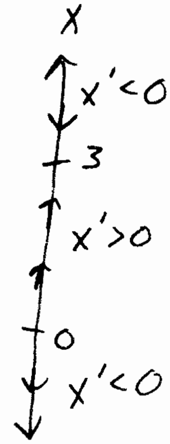
# Section 2.2

(4) $\dfrac{dx}{dt} = 3x - x^2 = (3-x)x$




Critical points: $x=0,\ x=3$

From the phase diagram at the right, $x=0$ is unstable and $x=3$ is stable.

Solve: $\displaystyle\int \frac{1}{(3-x)x}\,dx = \int dt$

Partial fractions

$$\frac{1}{(3-x)x} = \frac{A}{3-x} + \frac{B}{x}$$

$$1 = Ax + B(3-x)$$

$$1 = (A-B)x + 3B$$

$$B = \tfrac{1}{3}, \quad A = \tfrac{1}{3}$$

$$\frac{1}{3}\int \frac{1}{3-x} + \frac{1}{x}\,dx = \int dt$$

$$-\frac{1}{3}\ln|3-x| + \frac{1}{3}\ln|x| = t + C$$

$$\ln\left|\frac{x}{3-x}\right|^{1/3} = t + c$$

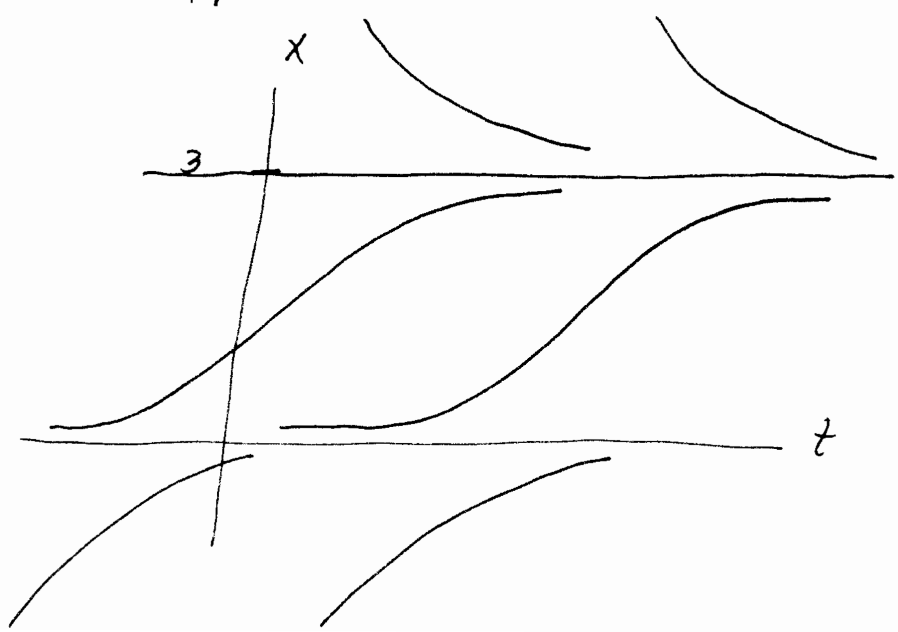
$$\left|\frac{x}{3-x}\right|^{1/3} = e^t e^c$$

$$\frac{x}{3-x} = Ke^{3t}$$

solving for $x$,

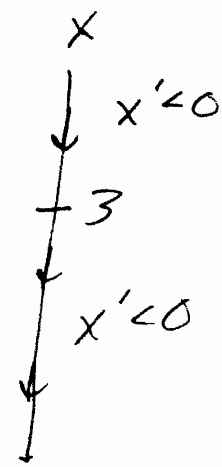
$$\boxed{x(t) = \frac{3Ke^{3t}}{1+\cancel{3}Ke^{3t}}}$$

Typical solution curves:



# Section 2.2

(8) $\dfrac{dx}{dt} = -(3-x)^2$




Critical point: $x=3$

$x=3$ is a <u>semi-stable</u> equilibrium solution.

Solve: $\dfrac{-1}{(3-x)^2}\,dx = dt$

$$\int -(3-x)^{-2}\,dx = \int dt$$

$$-(3-x)^{-1} = t+C$$

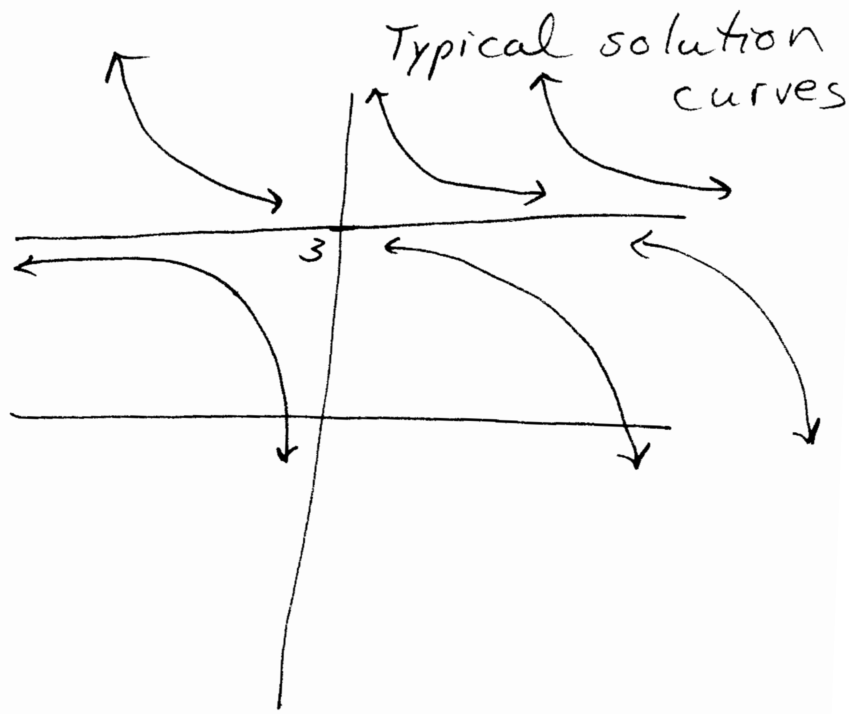
$$(3-x)^{-1} = -t-C$$

$$3-x = \frac{1}{-t-C}$$

$$\boxed{x = 3+\frac{1}{t+C}}$$

general solution

also $x(t)\equiv 3$ is a solution.



Section 2.2

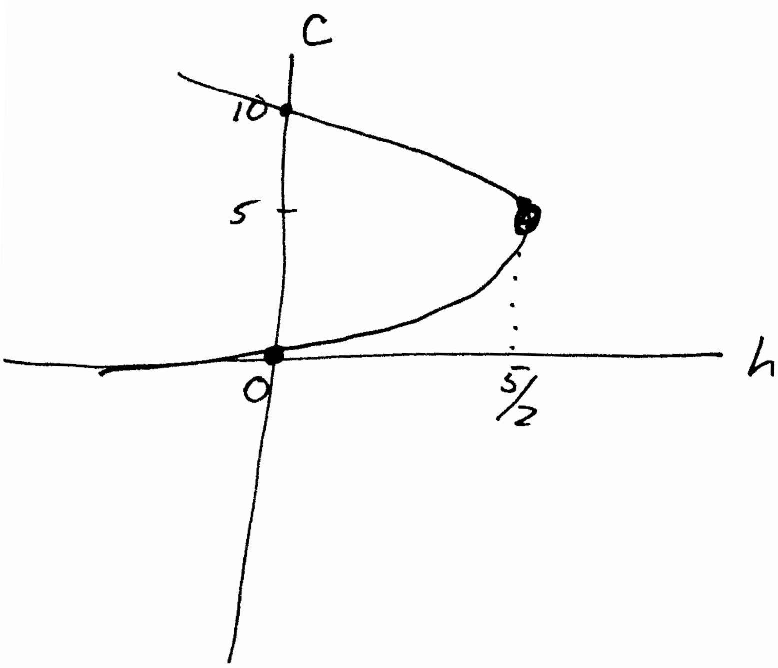
(19) $\frac{dx}{dt} = \frac{1}{10} x (10-x) - h$

Critical points are the values $x=c$ for which $\frac{dx}{dt}=0$. So

$0 = \frac{1}{10} c(10-c) - h$. Graph this to get the bifurcation diagram.

$h = \frac{-1}{10}(c^2-10c)$ Complete the square.

$h = \frac{-1}{10}(c-5)^2 + \frac{5}{2}$




The bifurcation point is $h = 5/2$.

If $h > 5/2$, there are no critical points.

If $h < 5/2$, there are 2 critical points.

## Section 2.4

(5) $y' = y - x - 1 \quad y(0) = 1$ Solution $y = 2 + x - e^x$

a. Step size $h = .0.25$

$f(x,y) = y - x - 1$

| | | |
|---|---|---|
| $x_0 = 0$ | $y_0 = 1$ (given) | $f(x_0, y_0) = 0$ |
| $x_1 = .25$ | $y_1 = 1 + .25(0) = 1$ | $f(x_1, y_1) = -.25$ |
| $x_2 = .5$ | $y_2 = 1 + .25(-.25) = .9375$ | |

b. Step size $h = .1$

$f(x,y) = y - x - 1$

| | | |
|---|---|---|
| $x_0 = 0$ | $y_0 = 1$ (given) | $f(x_0, y_0) = 0$ |
| $x_1 = .1$ | $y_1 = 1 + (.1)(0) = 1$ | $f(x_1, y_1) = -.1$ |
| $x_2 = .2$ | $y_2 = 1 + (.1)(-.1) = .99$ | $f(x_2, y_2) = -.21$ |
| $x_3 = .3$ | $y_3 = .99 + (.1)(-.21) = .969$ | $f(x_3, y_3) = -.331$ |
| $x_4 = .4$ | $y_4 = .969 + (.1)(-.331) = .9359$ | $f(x_4, y_4) = -.4641$ |
| $x_5 = .5$ | $y_5 = .9359 + (.1)(-.4641) = .88949$ | |

Approximation at $x = \frac{1}{2}$
with step size .25 : .9375
with step size .1 : .8895
actual value
$2 + .5 - e^{.5} \approx .8513$

$\boxed{\text{As expected, smaller step size results in better approximation}}$

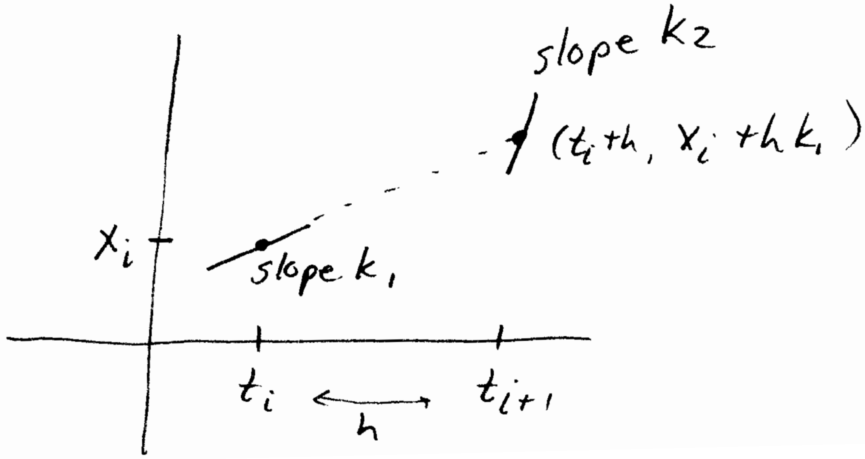
# Iode Project II

1.a. $k_1$ is the slope of the direction field at the point $(t_i, x_i)$.

$x_i + hk_1$ is the $x$ value we get by following the line with slope $k_1$, through $(t_i, x_i)$, a horizontal distance of $h$.

If we were doing the Euler method, $(t_i + h, x_i + hk_1)$ would be the next point in the approximation and would be called $(t_{i+1}, x_{i+1})$.

$k_2$ is the slope of the direction field at $(t_i + h, x_i + hk_1)$



(1b) Replace lines 6 and 7 from p. 7 of Lab II with the following 4 lines.

```
6   k1= feval (fs, tc(i), x )
7   k2= feval (fs, tc(i+1), x+h*k1 )
8   k = .5*(k1+k2)
9   x= x+ h*k
```

(3) Turn in your plots and answers.

(4) "  "  "  "