

HOMWORK 1 SUPPLEMENT — ALGEBRA, CALCULUS REVIEW

A little time spent now reviewing algebra and calculus facts will save you a lot of time later. So pull your calculus textbook off the bookshelf and do the following questions.

- (1) Suppose $y^{-1} = x + C$. True or false (circle one): is

$$y = x^{-1} + C^{-1}?$$

(If you're not sure, then try it with actual values for x and C .)

If false, then find a correct formula for y :

- (2) Notation: in mathematics “log” means the same thing as “ln”, in other words the natural logarithm or logarithm to base e .

Suppose $e^y = x + 1$. True or false (circle one): is $y = \log x + \log 1$? If false, then find a correct formula for y :

- (3) Suppose $\arcsin y = x + \pi/6$. True or false (circle one): is $y = \sin(x) + \sin(\pi/6)$? If false, then find a correct formula for y :

- (4) A quadratic equation $\boxed{ar^2 + br + c = 0}$ can be solved by factoring the equation as

$$a(r - r_1)(r - r_2) = 0,$$

in which case the roots are $r = r_1$ and $r = r_2$, or else by invoking the *quadratic formula*

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Advice. Since it is often difficult to see how to factor a quadratic equation, your best bet is usually to invoke the quadratic formula.

Here are the questions: find the roots of

(i) $2x^2 - 4x - 7 = 0$

(ii) $2x^2 - 4x + 7 = 0$

- (5) Consider a graph $y = f(x)$. Using that $\frac{dy}{dx} = f'(x)$ equals the slope of the graph at the point $(x, f(x))$ on the graph, explain with a picture why it is that $f'(x) \approx \frac{\Delta y}{\Delta x}$, where $\Delta y = f(x+h) - f(x)$ and $\Delta x = (x+h) - x = h$, and h is small.

[Recall $\frac{\Delta y}{\Delta x}$ is called a *difference quotient*, and gives the “rise in y ” divided by the “run in x ”. The difference quotient justifies our interpretation of the derivative $\frac{dy}{dx}$ as a rate of change of quantity y with respect to changes in quantity x .]

- (6) Evaluate the following derivatives and antiderivatives (indefinite integrals):

$$\begin{array}{ll} \frac{d}{dx} x^3 & \int x^4 dx \\ \frac{d}{dx} x^{-3} & \int x^{-4} dx \\ \frac{d}{dx} \sin x & \int \sin x dx \\ \frac{d}{dx} \cos x & \int \cos x dx \\ \frac{d}{dx} e^x & \int e^x dx \\ \frac{d}{dx} e^{-x} & \int e^{-x} dx \\ \frac{d}{dx} e^{2x} & \int e^{2x} dx \\ \frac{d}{dx} \log x & \int \log x dx \\ \frac{d}{dt} \frac{1}{t} & \int \frac{1}{t} dt \\ \frac{d}{dt} \frac{1}{t^2} & \int \frac{1}{t^2} dt \end{array}$$