Name:_____

Math 402 - Test #1 - February 9, 2007

Time: 50 minutes. Write your answers on the blank paper provided. Start a new page for each problem and be sure to number the problems. You may not use any books or notes. There are 100 points possible.

1. (20 points) State Euclid's five Postulates (axioms). Also explain briefly which one of these postulates is false for hyperbolic space.

2. (20 points) Based on your experimental work with spheres, give three differences between Euclidean geometry and spherical geometry. This question is very broad and many correct answers are possible, but please explain fully enough to make it clear to me what you are saying.

3. (5 points each part)

   (a) Define what is meant by a **consistent** axiomatic system.

   (b) Define what is meant by an **independent** axiom in an axiomatic system.

   (c) Define what is meant by a **complete** axiomatic system.

4. Consider the following axiomatic system:

   A1 There exist exactly three points.

   A2 Given any two points, there is exactly one line on which both points lie.

   A3 Given any point, there are exactly two lines on which the point lies.

   (a) (10 points) Give a model of this axiomatic system in order to show that the system is consistent.

   (b) (10 points) Prove that this axiomatic system is not complete. Be sure to explain your reasoning fully, using the definition of complete.

5. (10 points) Explain why, in an axiomatic system, some of the terms must be undefined terms.

6. (15 points) Consider the following axiomatic system which we studied in class:

   A1 There are exactly three students.

   A2 For every pair of students, there is exactly one class in which they are both enrolled.

   A3 Not all of the students belong to the same class.

   A4 Two separate classes share at least one student in common

   Prove that two separate class share one and only one student in common. (You should do a proof from the axioms, not from a model. Be sure to say which axioms you are using.)