Name:_____

Math 402 - Test #3 - April 9, 2007

Time: 50 minutes. Write your answers on the blank paper provided. Start a new page for each problem and be sure to number the problems. You may not use any books or notes. There are 100 points possible.

1. (7 points each part) Give the definition of each of the following. You don't need to give examples or additional properties, just the definition.

   (a) A *fixed point* P of an isometry f.

   (b) *Reflection.*

   (c) *Omega point.* Your definition should not refer to any particular model.

2. (20 points) Give an outline of the proof that every isometry can be written as the composition of three or fewer reflections.

3. (15 points) Give the definition of Saccheri quadrilateral and prove that the summit angles of a Saccheri quadrilateral are congruent.

4. (15 points) Prove that for any omega triangle $PQ\Omega$, the sum of the angles $\angle PQ\Omega$ and $\angle QP\Omega$ is always less than 180 degrees.

5. (5 points each part) For parts (a), (c) and (d) below, one sentence is enough.

   (a) What is hyperbolic geometry (sometimes called non-Euclidean geometry)?

   (b) List the axioms of hyperbolic geometry.

   (c) The inventors of hyperbolic geometry in the 1700s and 1800s did not set out to create a new geometry. Instead, they were trying to do what?

   (d) Why were the inventions of models such as the Poincare model and the Klein model important in the history of non-Euclidean geometry?

6. (3 points each part) Answer True or False for each part. No explanation is needed and there is no partial credit.

   (a) If an isometry f can be written as the composition of six reflections, then f can also be written as the compositon of two reflections

   (b) An isometry with exactly one fixed point must be a rotation.

   (c) Given two distinct point P and Q, there is exactly one isometry f with the property that $f(P) = Q$.