Name:_____

Math 402, Section C13 - Test #3 - April 19, 2006

Time: 55 minutes. Write your answers on the blank paper provided. Be sure to number the problems. You may not use any books or notes (except for those provided) or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (a) (10 points) Give the definition of Saccheri quadrilateral.

   (b) (15 points) Prove that the summit angles of a Saccheri quadrilateral are always congruent.

2. (15 points) Non-Euclidean geometry was first invented by mathematicians who were trying to prove *what* about Euclidean geometry? Answer in a few sentences.

3. (20 points) Given a reflection r across a line l and a translation T in the same direction as l, prove that
$$r \circ T = T \circ r.$$
   Hint: work in (x, y)-coordinates, choosing coordinates in such a way as to make your computations simple.

4. (10 points) Describe the invariant lines of a glide reflection. (You do not need to give a proof, but describe clearly which lines are invariant.)

5. (30 points) For each of the following, answer true or false and give a brief explanation of your answer (just a sentence or two, not a formal proof, but some explanation why you answered true or false.)

   (a) The composition of any two rotations (in Euclidean geometry) is a rotation.

   (b) In hyperbolic geometry, any two lines which do not intersect are called "limiting parallels."

   (c) In Euclidean geometry, the set of all rotations and translations forms a group of symmetries. (See definition below)

Definition: A *group of symmetries* is a set of Euclidean isometries that have the following properties:

1. Given any two elements of the set, the composition of the two elements is again a member of the set.
2. The compositin of elements is an associative operation.
3. The identity is a member of the set.
4. Given any element of the set, its inverse exists and is an element of the set.