Name:_____

Math 402 - Final Exam - May 8, 2007

Time: 3 hours. Write your answers on the exam. You may not use any books or notes other than the page of theorems provided. There are 100 points possible.

1. (5 points) Give the definition of *isometry*.

2. (10 points) Prove that if f and g are isometries, then their composition $f \circ g$ is also an isometry.

3. (10 pages) List all the types of isometries.

4. (5 points each part)

   (a) Define what is meant by a **consistent** axiomatic system.

   (b) Define what is meant by an **independent** axiom in an axiomatic system.

   (c) Define what is meant by a **complete** axiomatic system.

5. (10 points) Prove that the following axiomatic system is not complete.

"Page", "word" and "on" are undefined terms in this axiomatic system.

A1 There are exactly three pages.

A2 For any two distinct pages, there is exactly one word which is on both pages.

A3 Each word is on at least two pages.

6. (5 points each part) How, if at all, does hyperbolic geometry differ from Euclidean geometry in each of the following categories?

   (a) parallel lines

   (b) Side-Angle-Side Congruence Theorem for triangles

   (c) similar triangles

7. (10 points) Is it possible or not to tile (tessellate) each of the following spaces with equilateral triangles with eight triangles meeting at each vertex? Explain briefly.

   (a) Euclidean space

   (b) hyperbolic space

   (c) the sphere

8. (10 points) Recall that in hyperbolic space, there is a constant k such that for any triangle,

$$\text{area}(\triangle ABC) = k^2 \text{defect}(\triangle ABC).$$

Use this equation to find numbers m and M such that

$$m \leq \text{area}(\triangle ABC) \leq M$$

for *all* triangles in hyperbolic space. Is there any $\triangle ABC$ which actually has area m or M? Explain.

9. (3 points each part) Answer true or false. No explanation is needed and there is no partial credit.

(a) If an isometry has two distinct fixed points, then it must be the identity.

(b) In Euclidean geometry, Euclid's 5th postulate is an independent axiom.

(c) If an isometry can be written as the composition of five reflections, then it reverses orientation.

(d) Hyperbolic geometry is a *consistent* axiomatic system.

(e) Given any two parallel lines l and m in hyperbolic space, there is a third line n such that $n \perp l$ and $n \perp m$.